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Journal

of

THE ROYAL AERONAUTICAL SOCIETY

JUNE 1954

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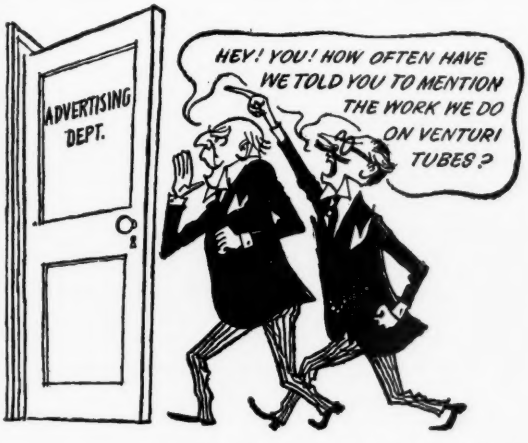

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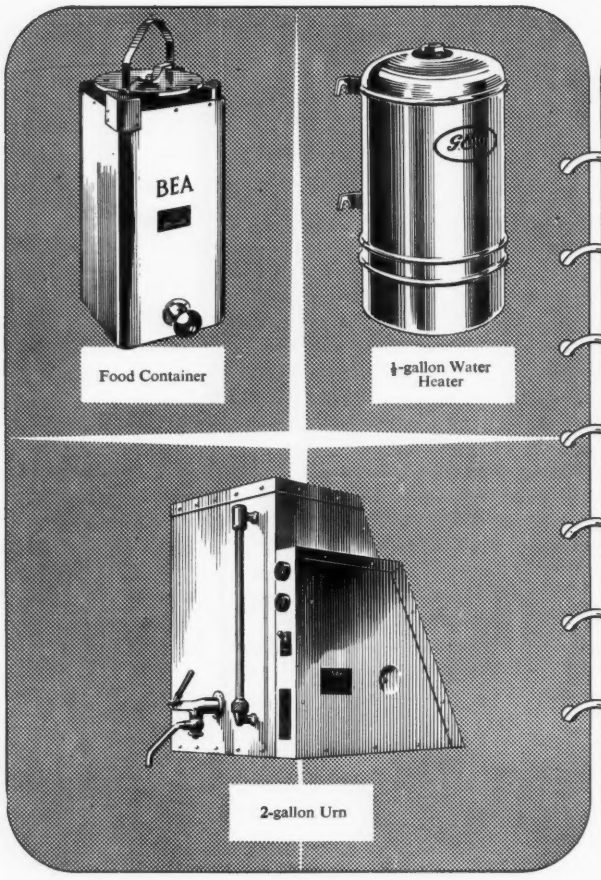

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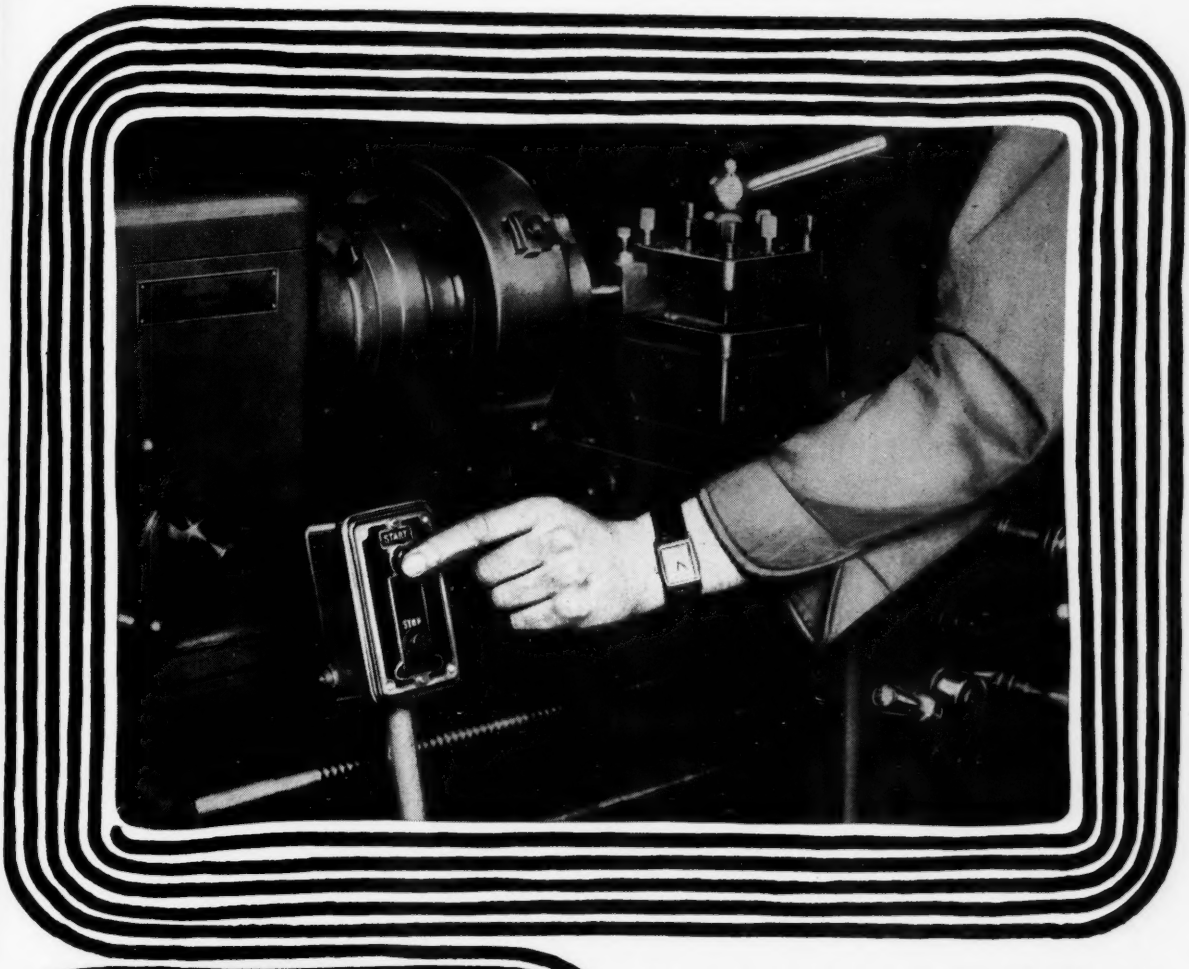

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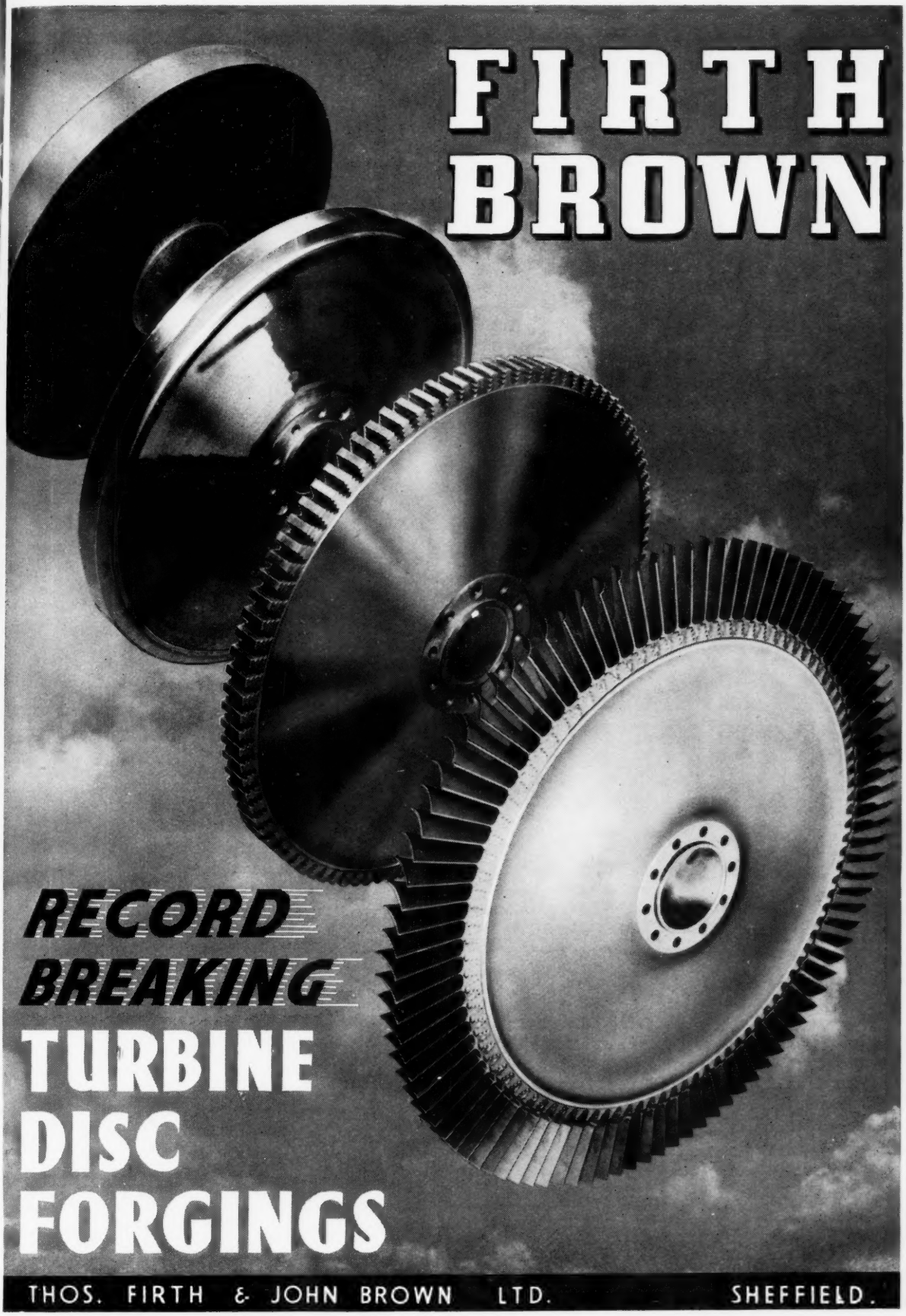
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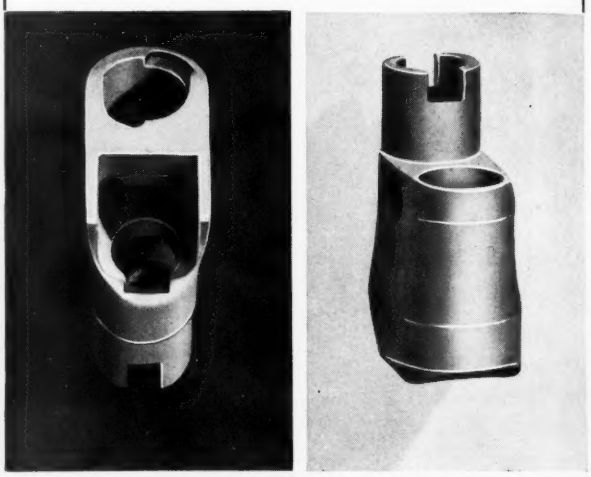

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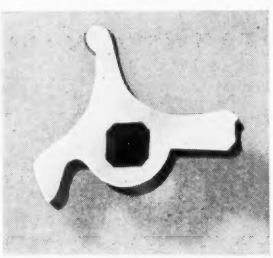

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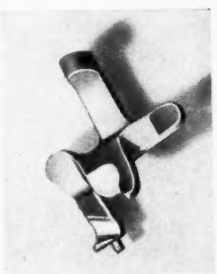

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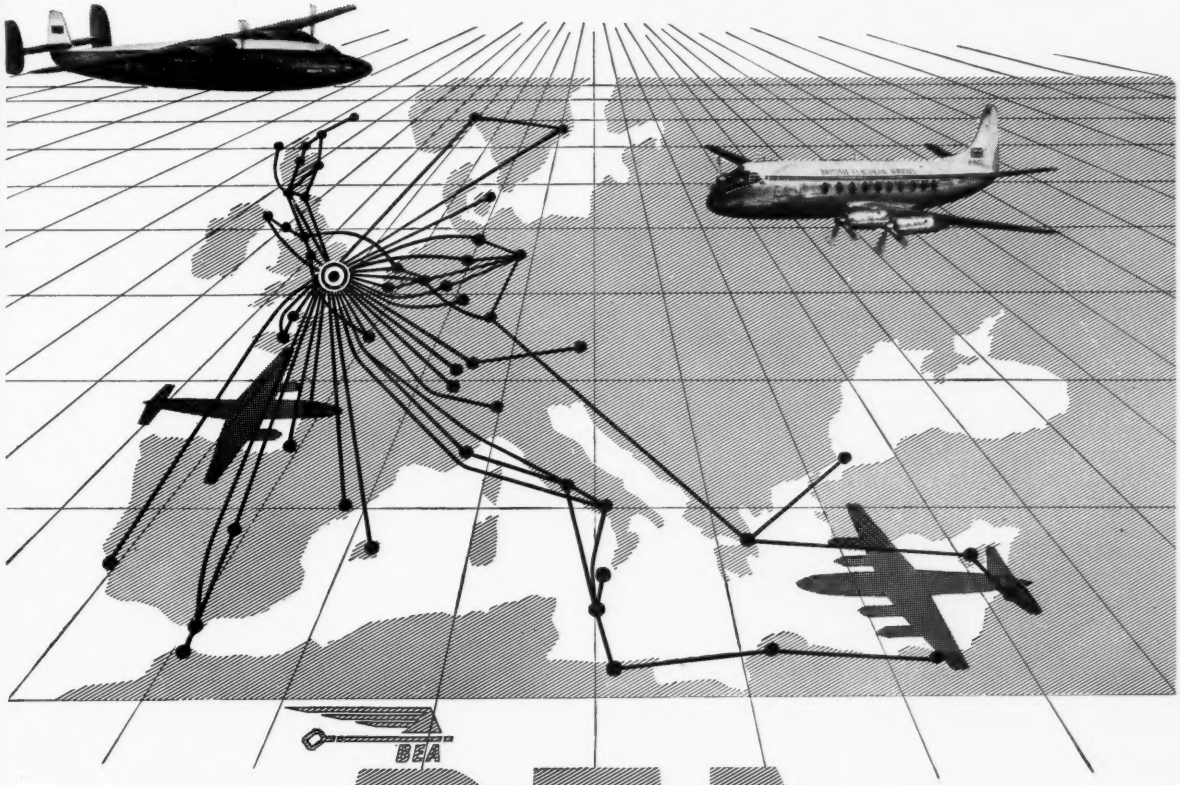

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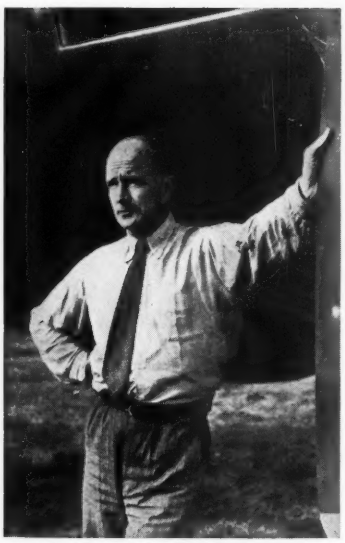

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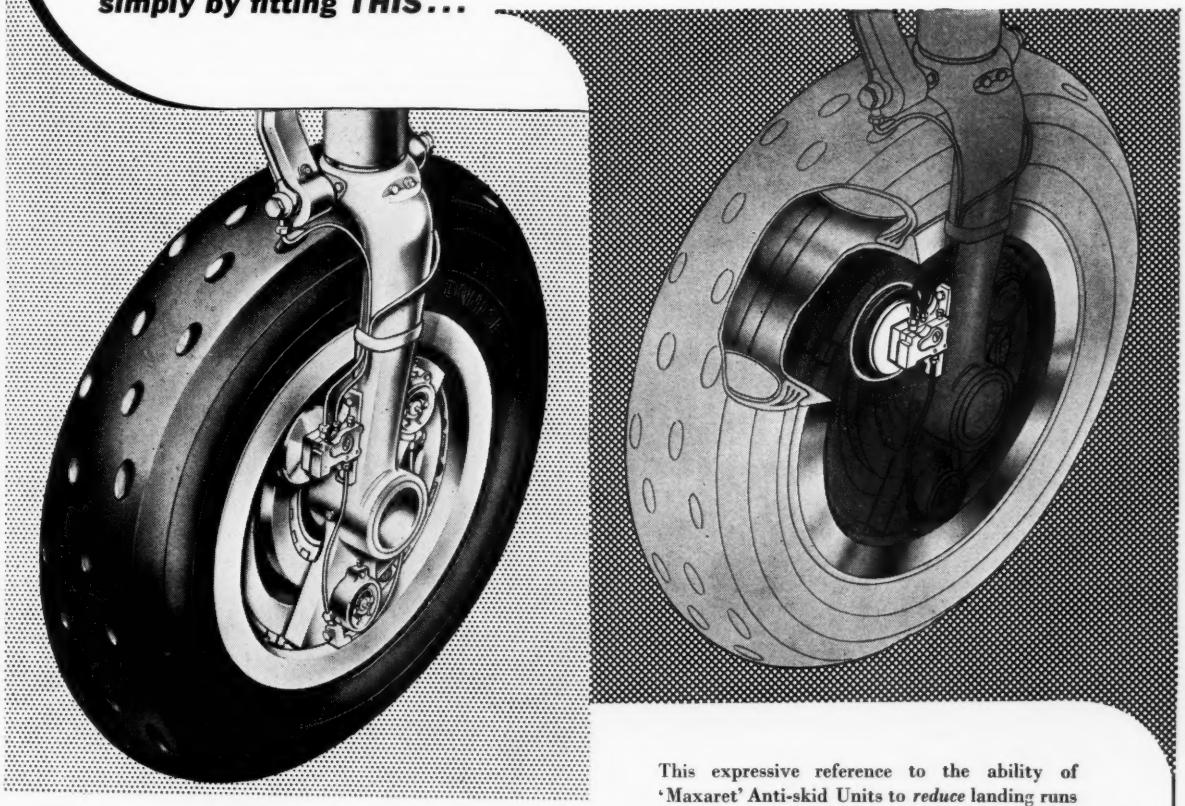

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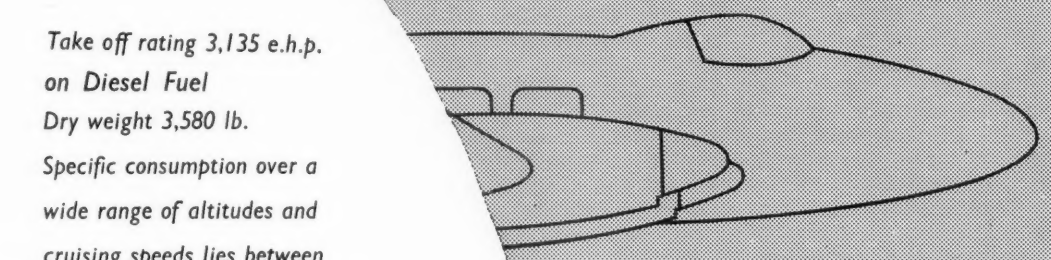

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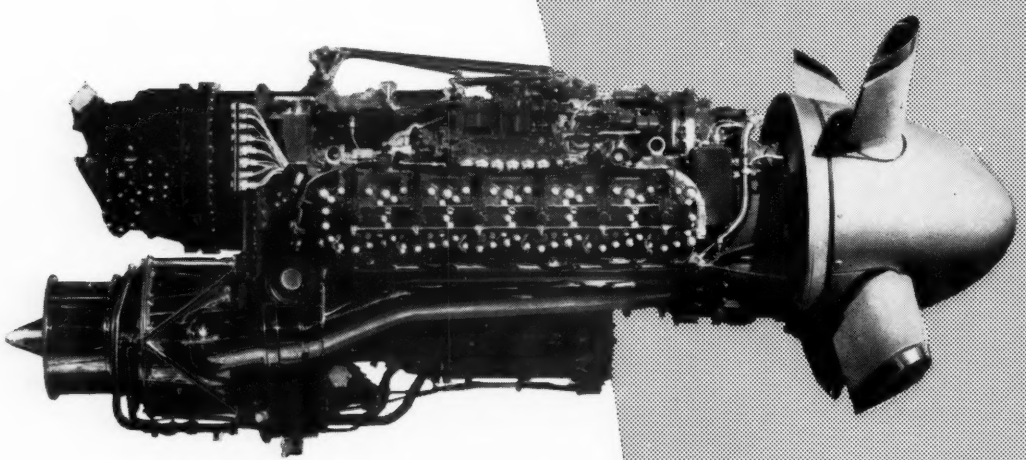

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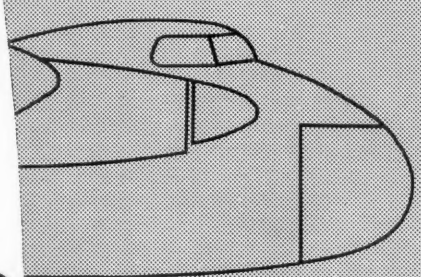

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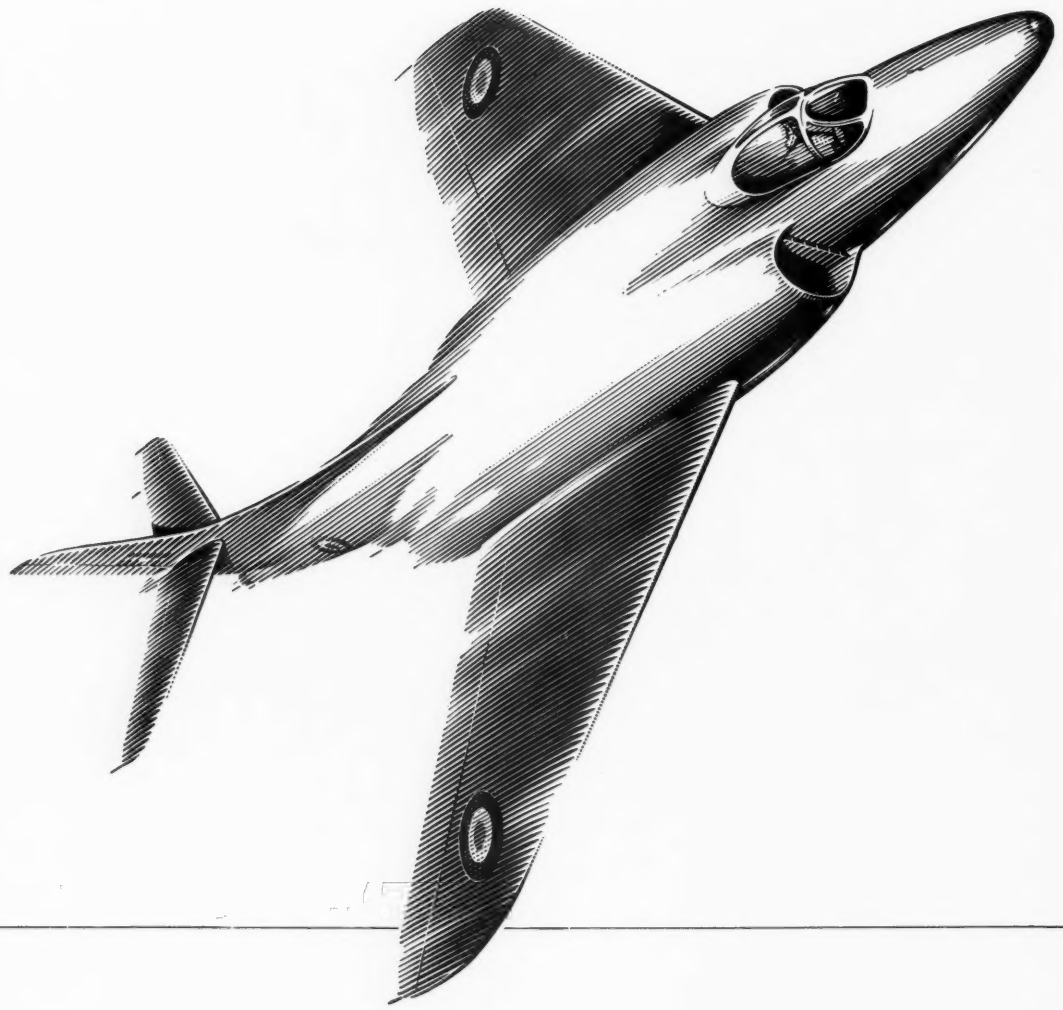

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SQUADRON LDR. H. G. A. SCILLEY, A.F.R.Ae.S.
B. S. SHENSTONE, M.A.Sc., F.R.Ae.S.
W. TYE, O.B.E., B.Sc., F.R.Ae.S.
DR. P. B. WALKER, M.A., F.R.Ae.S.

OFFICERS

Honorary Treasurer: MAJOR G. P. BULMAN, C.B.E., B.Sc., F.R.Ae.S.
Solicitor: L. A. WINGFIELD, M.C., D.F.C., A.R.Ae.S.
Secretary: DR. A. M. BALLANTYNE, T.D., A.F.R.Ae.S.

**The Vice-Presidents are elected at the May Meeting of Council.*

NOTICES

An annual sum of £250 is available for premium awards for papers, including Technical Notes, published in the Journal. Members and non-members of the Society are invited to submit papers on any aspect of aeronautics

Election of New President

Sir Sydney Camm, C.B.E., F.R.Ae.S., has been elected President of the Society for the year 1954/5 and took office at the Annual General Meeting on the 6th May 1954.

Council for 1954/5

The following are the newly-elected Members of Council:

Professor W. J. Duncan, C.B.E., D.Sc.(Eng.), F.R.S., M.I.Mech.E., F.R.Ae.S.
P. A. Hearne, D.C.Ae., D.L.C., Grad.R.Ae.S.
Squadron Leader H. G. A. Scilley, A.F.R.Ae.S.
Dr. P. B. Walker, M.A., F.R.Ae.S.

The following were re-elected to the Council:
Air Commodore F. R. Banks, C.B., O.B.E., F.R.Ae.S.
Major G. P. Bulman, C.B.E., B.Sc., F.R.Ae.S.
Dr. E. S. Moult, Ph.D., F.R.Ae.S.

Annual General Meeting

The Annual General Meeting of the Society was held at 4 Hamilton Place, W.1, on 6th May 1954, at which the Annual Report and Balance Sheets of The Royal Aeronautical Society and Aeronautical Trusts Ltd. for 1953 were approved. The Complete Report and Balance Sheets were published in the April 1954 Journal.

New Fellows

The following elections to Fellowship were announced at the Annual General Meeting on 6th May 1954:

C. Abell	G. Gabrielli	C. F. Joy
L. R. E. Appleton	W. G. Glendinning	W. A. Mair
A. C. C. Orde	R. Hafner	E. B. Moss
J. S. Clarke	W. S. Hemp	S. Neumark
C. L. Dann	H. L. Hibbard	L. F. Nicholson
J. T. Dyment	W. G. Jennings	W. A. Tamblin
E. Fielding	W. P. Jones	H. C. B. Thomas
	R. H. Woodall	

Society of British Aircraft Constructors' Educational Grant

The attention of members is drawn to the Society of British Aircraft Constructors' Educational Grants, administered by the Society.

The Grants are designed for the assistance of young men who are unable, for financial reasons, to obtain training in aeronautical engineering. All holders of S.B.A.C. Grants are expected to qualify for a technical grade in the Royal Aeronautical Society.

Applicants should be between the ages of 16 and 18 on 1st September in the year of the award. The closing date for applications for this year is 30th June 1954. Application forms may be obtained from the Secretary.

Graduates' and Students' Section

A visit has been arranged to the Headquarters of the London Control Zone on Saturday 26th June. Visitors will be able to see the equipment, etc., connected with Air Traffic Control at Uxbridge and London Airport. As numbers are strictly limited members are requested to apply, as soon as possible, to the Honorary Visits Secretary, D. J. W. Richards, 18 Gevena Road, Kingston, Surrey.

Garden Party, 13th June 1954, London Airport

An application form for tickets for the 1954 Garden Party has been sent to each Member with a registered address in the United Kingdom. Tickets, which include tea and car park, are 10s. each for adults and 5s. each for children under fifteen years of age.

Although there is no flying programme as in former years, Members will have an opportunity to see the working of London Airport and to see at close quarters airliners taking off for, and landing from, many parts of the world. There will be a number of the modern airliners available for inspection—a Vickers Viscount of British European Airways, a Lockheed Super Constellation of Trans-Canada Airlines, and a Stratocruiser of British Overseas Airways. In addition Members will see for the first time the Nash Collection of Veteran Aircraft recently purchased by the Society. Special buses will take Members and their guests on a tour of the airport and the afternoon should prove a very interesting one.

Members who have not yet applied for tickets should do so quickly.

News of Members

Z. M. Ciolkosz (*Associate Fellow*) has been awarded the Wright Brothers Medal, jointly with Mr. D. N. Meyers, by the Institute of the Aeronautical Sciences, for their paper on "Matching the Characteristics of Helicopters and Shaft Turbines."

A. J. Clark (*Associate Fellow*) has now left the County Technical College, Guildford, to become Senior Lecturer in Mechanical Engineering, at the Poplar Technical College.

W. Gillespie (*Associate Fellow*) has recently been appointed Principal of The College, Swindon.

R. Hadekel (*Associate Fellow*) has been appointed Engineering Superintendent (Hydraulic) to the Sperry Gyroscope Co. Ltd.

V. T. Kilburn (*Associate*), of the Southern Africa Division, has taken up an appointment with the Electricity Supply Commission.

W. McKim (*Associate Fellow*) has been appointed to the Professional Engineers grade in the Ministry of Supply as an Eng. III, and has been posted to "Process Research" at R.A.F., Chorley.

Squadron Leader F. L. Payne (*Associate Fellow*) has left the M.O.S. National Gas Turbine Establishment, Pyestock, to take up an appointment with the Air Ministry as Technical Superintendent (Engineer) at No. 7 M.U., R.A.F. Quedgeley, Gloucester.

J. Woodhouse (*Associate Fellow*) has joined the staff of Hedley S. Crabtree and Company Ltd., Consultant Engineers.

Graduates' and Students' Section

A visit has been arranged to the National Physical Laboratory on the morning of Saturday 17th July. Members are requested to apply to the Hon. Visits Secretary, D. J. W. Richards, at 18 Geneva Road, Kingston, Surrey, as soon as possible.

Elections

Associate Fellows

Arthur Ronald Boardman (*from Associate*)
Alfred George Campion (*from Graduate*)
William Arthur Crago (*from Student*)
William Richard Cushing
John Brian Helliwell (*ex-Student*)
Anthony Herbert-Caesari (*from Graduate*)
James Ivor James (*from Graduate*)
Lancelot Elworthy Jarman
Augustine Patrick Kennedy
Kenneth David Little (*from Graduate*)
Clive Nelson
Donald McKay Ridland (*from Associate*)
Richard Michael Sellens
William Charles George Smith (*from Associate*)
William Edward Walker
Peter Charles Harford White (*from Graduate*)
Archibald William Alfred Prott Wilson (*from Graduate*)
Henry Hok-Yong Wong (*from Graduate*)

Associates

John Llewellin Cox (*from Student*)
William Marshall Dunn (*from Graduate*)
Alfred Lewis Gilder
Bryan Edward John Heard
Arthur Milner Dickeson Howes
Albert Wallace Laughton (*from Companion*)
Eric Lumley
Sidney Charles Maultby

Graduates

Shaikh Tabarak Ali
James Arnold Forrest Allan (*from Student*)
Eric Fell
Vishnu Anant Gokhale
Matthew Alexander Hutchison (*from Student*)
Kishor Kanayalal Kirtikar
Geoffrey Francis Langdon (*from Student*)
William Andrew Lemmer (*from Student*)
Vernon Roy Parkhouse
William Alan Pollard (*from Student*)
Derek Jess Wilton Richards (*from Student*)
Richard William Roberts

Students

Ronald Boardman
William Robert Bryce
Graham Edward Cook
John Frederick Coplin
Michael Charles Frengley
Leslie Arthur Hall
Donald Stuart Lawson
Arthur John Murray
Geoffrey Stott
David Ralph Swanston
Walter Edward Turner
John David Warrin

Companions

Phillip Charles Fairfax Morgan
William Philip Warner

Changes of Address

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month in order to be effective for the* Journal for the following month.

Diary

BRANCHES

June 21st

Halton—Operational Rôle of the R.A.F. in the Second World War. Wing Commander Vivian Varcoe. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

June 23rd

Halton—Visit to the "Esso" Refinery at Fawley.

June 29th

Singapore—Lecture: Aircraft Instruments. Shell House. 8.30 p.m.

July 14th

Halton—Visit to the B.O.A.C. Maintenance Base, London Airport.

July 27th

Singapore—Lecture: Radar. Shell House. 8.30 p.m.

Journal Binding

Self-Binder Cases

Self-Binder cases of the "Easibind" type are available from the offices of the Society. These binders are for members who do not have their Journals permanently bound, or who wish to keep their Journals together during the year for binding later.

These cases will hold 12 Journals which are kept in place by means of flexible steel wires. Journals can be inserted or withdrawn easily without damage, so preserving the contents for permanent binding later. The Journals will open flat at any page.

The binder is strongly made in durable dark blue leather cloth on stiff board covers and has gold lettering on the spine. The year is not blocked on the spine but there is a panel on which members who wish to use the binder as a permanent case can put the date.

The cost is 11s. 6d. each including postage and packing for either the size to fit 1952 and previous Journals, or for the size to fit the Journal from January 1953, which has been increased in size. Orders and remittances should be sent direct to the Secretary at the Offices of the Society and it is *important to state whether the old size or new size is required.*

Permanent Binding

There is no increase in the price of permanent binding of Journals. The prices are:—

1953 Volume (including packing and postage) 16s. 0d.
Previous Volumes (including packing and postage) 18s. 0d.

Journals, *with a note of the name and address of the sender,* should be sent direct to The Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 58 JUNE 1954 NUMBER 522

THE FIRST BARNWELL MEMORIAL LECTURE

The 902nd Lecture to be given before the Society, which was also the 15th Main Lecture at a Branch of the Society and the First Barnwell Memorial Lecture, was held under the auspices of the Bristol Branch on 4th March 1954. The Lecture was delivered by Major G. P. Bulman, C.B.E., B.Sc., F.R.Ae.S., Director of Construction and Research Facilities, Ministry of Supply and a Past President of the Society.

Dr. A. E. Russell, F.R.Ae.S., President of the Bristol Branch, opened the meeting and then invited Sir William Farren, C.B., M.B.E., M.A., F.R.S., M.I.Mech.E., F.R.Ae.S., President of the Society, to preside.

DR. A. E. RUSSELL: This evening is an important occasion for at least three reasons. The first is that this is the second occasion on which the Bristol Branch has had the privilege of acting as hosts at a full meeting of the Royal Aeronautical Society. The second reason is that we are celebrating the twenty-fifth anniversary of the formation of the Bristol Branch. The third reason, and the most important, is that we are gathered to listen to a lecture, the first in a new series of Memorial Lectures, to the late Captain Barnwell, who was one of the early pioneers in aircraft development and a man whom many of us remember with affection and respect.

As hosts we extend a sincere welcome to our guests, many of whom have travelled considerable distances to pay tribute to the late Captain Barnwell.

I do not wish to mention many names, but I feel that I must mention Air Commodore H. Busteed, who was the first Test Pilot to fly the aircraft designed by Captain Barnwell in the early days. We have present many representatives from Yeovil, Gloucester and Cheltenham.

The very first lecture read before the Bristol Branch was by Captain Barnwell himself, and it was about the impressions he had gathered at the Chicago Aero Show in 1928. It is also interesting to recall that the early days of this Branch were very precarious, and had it not been for the efforts of Captain Barnwell and Mr. Tinson the Branch might have fallen, at least temporarily, out of service.

It is my great pleasure to introduce to your Sir William Farren, President of the Royal Aeronautical Society.

SIR WILLIAM FARREN: Before I come to my main task of introducing the Lecturer I want to take the opportunity of saying a few things, since to the Council of the Society a Main Lecture is an occasion. It is sometimes our custom to say something about the Society generally and it is particularly appropriate to do so when not in London. The idea that everything that is worth while happens in London is wrong; the great body of the membership of the Society is spread around the country and indeed, there is a considerable part of it outside this country. It is from the members in the country that we can expect the new blood which will keep the Society going in the future.

It is useful to point out the objects of the Society and I think the words on the Membership diplomas describe them better than any others:—

"The Society is established for the general advancement of aeronautical science and engineering and more particularly to promote that species of knowledge which distinguishes the profession of aeronautics."

I draw your attention particularly to the words "science" and "engineering." Sometimes we are rather shy of the word "science" as though it were something irregular, but we should be proud to use those two words to sum up the object of our existence.

Those who are full Members, or are in any grade of membership of the Society, must have certain qualifications and it is the duty of the Council to ensure that the status of those grades is in no way allowed to fall, and to ensure that, we have to work to certain rules. I find that our motives are sometimes misinterpreted by those who feel that they should be in a higher grade than the Council is prepared to admit. Although we have a certain latitude in intepreting the Rules, we take the view that the worst thing for the Society is to let the qualifications go down.

One of the great strengths of any Society such as ours is its general publications, if for no other reason than that it is the only medium by which members can keep in touch with the Society's activities. Besides the JOURNAL we have another publication, THE AERONAUTICAL QUARTERLY, and we have now decided that it should again be published quarterly; I would draw the attention of all members to the value of it and ask them to support it, by writing, if they can, papers. We are particularly anxious that there should be a considerable proportion of papers dealing with the experimental side of aeronautics and I hope you will contribute to this publication. We want to make it—what I am sure it could be—the foremost journal of its kind.

Dr. Russell has mentioned that this is the 25th anniversary of the Bristol Branch. I think you started with about a hundred members and have now about 450; approximately half your membership is composed of full members of the Society in one grade or another, and rather less than half constitutes that supply of new blood from which I hope our new membership will come, in this case from the Bristol area. A Branch of 450 is quite substantial, one of our largest. On behalf of the Council I compliment you on your activities.

This is the fifteenth Main Lecture which has taken place outside London. I have a feeling that we could, with advantage increase the number in due time, but we must do it with care. We feel that one of the great opportunities for a Main Lecture in the provinces occurs when the Branch in question associates the lecture with the name of someone who was a member of that Branch and who was of great distinction. My old friend Captain Barnwell is an ideal example.

Your lecturer this evening is an ex-President of the Society, Major Bulman, and I do not think there is anyone better qualified to tell you about Captain Barnwell. They were great friends and I now ask Major Bulman to give the Barnwell Memorial Lecture.

Captain F. S. Barnwell

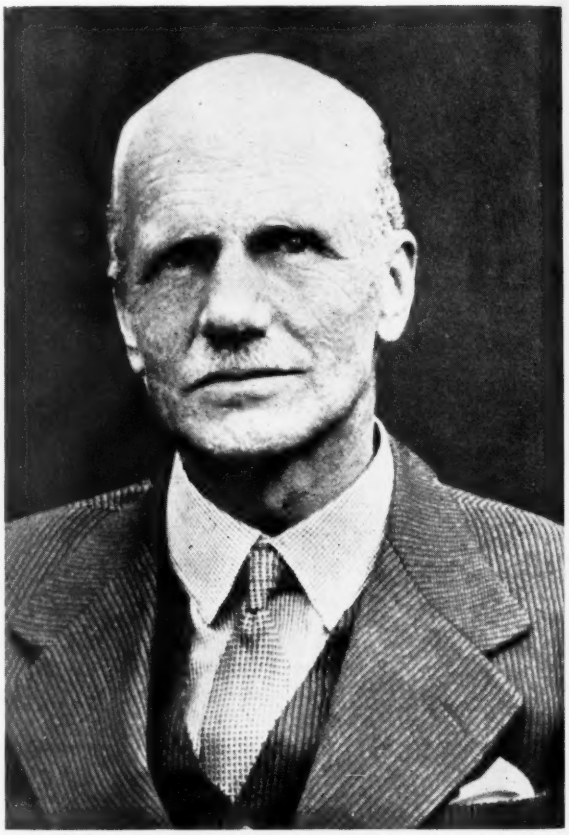
by

MAJOR G. P. BULMAN, C.B.E., B.Sc., F.R.Ae.S.
(Director of Construction and Research Facilities, Ministry of Supply)

FOR US in the early evening of life, who have witnessed the development of aviation since the Wright brothers flew in 1903, for us, it is hard—and a little sad—to realise that some of our great pioneers are already becoming almost mythical characters. They were our personal friends with whom we worked most happily; yet, to the young men of today, who are now carrying, or are about to don, high responsibilities in the aeronautical world, they are indeed just names. And so it is fitting that the Royal Aeronautical Society should have inaugurated these Memorial Lectures, of which this is the third. I am deeply sensible of the honour and responsibility which have fallen to me by the choice of the Bristol Branch to pay this first tribute to Captain Barnwell. May I say at once how indebted I am for so much of the detail of this appreciation to Mr. C. W. Tinson, who served with Barnwell for most of his career at Bristol; to Mr. C. H. Barnes, who has written the history of the Bristol Aeroplane Co.; and several other "old hands." It is significant that in all my conversations with them, they referred always with obvious affection and deep respect to "the old man."

Frank Sowter Barnwell, born in 1880 in South East London, moved with his parents a year later to Glasgow. After four years at Fettes, the Eton (or is it Harrow?) of Scotland, he served six years apprenticeship 1898-1904 with the Fairfield Shipbuilding Co. at Govan, in which his father was a partner. At the same time he attended evening classes at the Technical College and spent the winter months from 1900 to 1905 at Glasgow University, obtaining his B.Sc. Degree in Naval architecture in 1905. After a year in America, spent in the hull draughting department of a shipbuilding firm, he returned in 1907 to join his brother Harold and a younger brother in their own small firm, the Grampian Engineering and Motor Co. Ltd., at Stirling. There, following their earlier construction in 1905 of two extremely home-made gliders, and an aeroplane fitted with a Peugeot motor cycle engine, without much success except in experience in "how not to build," the brothers constructed in 1908 a Parasol monoplane with a two-cylinder air-cooled Vee engine designed by Harold. This was followed in 1909 by a bi-plane with a Humber T.T. car engine with twin chain-driven pusher propellers, which crashed on its second flight; and then a Tractor monoplane with a 40 h.p. twin-cylinder water-cooled engine designed by Harold which, in 1910, won a £50 prize given by the Scottish Aeronautical Society.

Early in the same year, 1910, Frank became engaged to the daughter of Lt. Col. Charles Sandes of Stirling and reverted, no doubt for financial reasons, to shipbuilding. Harold migrated to Brooklands where he got his ticket on a Bristol biplane in 1912 and soon after

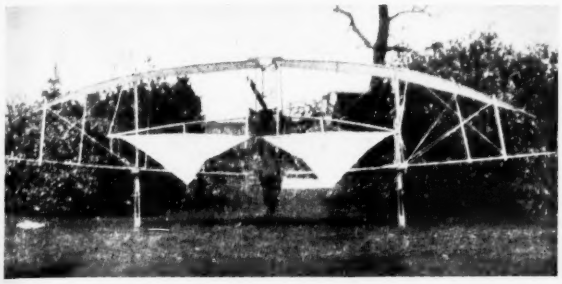


Captain F. S. Barnwell.

joined Vickers as test pilot. I myself had my first flight in May 1915 with Harold in a Vickers Gun Bus. Not long after, having returned to duty soon after a serious attack of 'flu, he collapsed in the air, crashed, and was killed.

In the meantime Frank, now married, came back to aviation and after glancing at the Avro Company in Manchester took up an appointment in Bristol as a draughtsman in March 1911 with the then British and Colonial Aeroplane Co. This Company had been founded in 1910 through the vision and courage of Sir George White, first Baronet, with a capital of £25,000, as an offshoot of the Bristol Tramway Co. at Brislington, in which Sir George had already established himself as a great pioneer in Transport in the replacement of horse-drawn trams with electric traction.

He had watched flights during a holiday in Cannes in 1909 and was immediately convinced of the vast potential value of flying as a means of transport. The

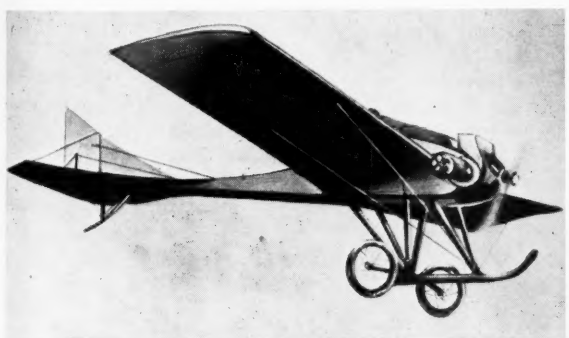


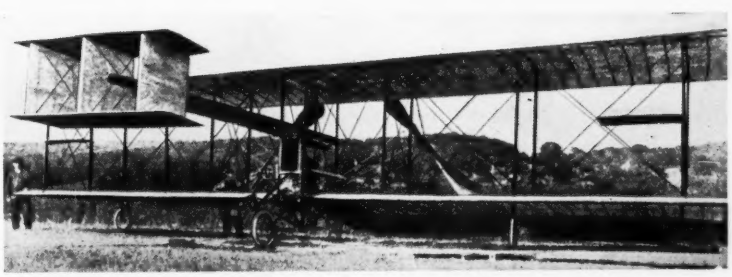
The Barnwell machine of 1905 with a Peugeot motor cycle engine.

new company (whose capital had been increased tenfold by 1913) had already established the highest reputation in the quality of its design and superlative workmanship, its enterprise in seeking foreign markets, and in the efficiency of its Flying Training Schools.

Design work, as was common in those days, was shared among several:—Challenger the Chief Engineer and Works Manager, Grandsiegne, Pierre Prier, Gordon England, and Harry Busteed, the last four being also pilots; and later by Coanda, son of the then Rumanian War Minister. The aircraft were built partly in the Brislington works and then in buildings erected on the 2½ acres of land bought by Sir George adjoining Filton House.

Barnwell's first job at Bristol, with Tinson as his leading draughtsman, was on a project initiated by Lt. C. D. Burnie, R.N. (who later, as Sir Dennis, was largely concerned in the construction of the airship R.100 in the late 1920's); this was the conversion of one of Gordon England's biplanes as a Naval aircraft fitted with hydrofoils or hydropeds and a water propeller driven by a Canton Unnè water-cooled radial engine in





(*Above*) The Barnwell monoplane which won the Scottish Aeronautical Society prize in 1910 and (*right*) the 1909 Barnwell biplane with a Humber T.T. engine.

Captain F. S. Barnwell.

conjunction with the airscrew. This adventure was financed by the Admiralty and known as X.1 and to ensure the necessary security Barnwell and Tinson were shut up in one of a row of cottages in Fairlawn Avenue which still stands within the Filton works area. The results achieved with this curious contraption were not very successful and came to an abrupt end in 1913, when the X.3 piled up on a submerged sandbank after being towed to assist its take-off by a destroyer which let go a little early. But this was no reflection on Barnwell who had been given little, if any, freedom of action on the basic design.

Barnwell was not absorbed into the main organisation under Coanda, Chief Designer since 1912, until the summer of 1914 but he had been given the fuselage of one of Coanda's monoplanes with authority to design a new aeroplane round it. The result was the prototype of the famous Bristol Scout, for which all the necessary design and stressing calculations were done by Barnwell

The "X" water-conversion aeroplane which was F. S. Barnwell's first job with the Bristol Aeroplane Company.

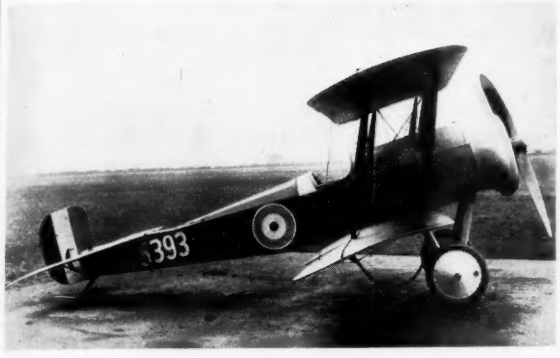
in a penny exercise book, a complete set of drawings prepared by him and Tinson in duplicate books, from which the carbon copies were sent to the shops. It had staggered wings of 22 ft. span, a simple two wheel undercarriage, double acting ailerons (a great advance), divided elevator, and a balanced rudder of light steel tubing. With its 80 h.p. Gnome rotary engine, a pilot, and fuel for three hours, it weighed 950 lb. At Larkhill, flown by Harry Busteed, that great Australian, in February 1914 it reached 97 m.p.h. with a climb of 800 ft. per minute. After exhibition at the 5th International Aero Show at Olympia in March its span was increased to 24 ft. 7 in. and in May it completed an official A.I.D. performance test at Farnborough, showing a speed range of 97·5 to 40 m.p.h.

Two replicas were built, designated Scout B, and delivered to the then Royal Aircraft Factory at Farnborough in August 1914. They soon went on to France nicknamed the Bristol Bullet. In one Captain Vessey Holt (who became D.T.D. in 1929) shot down a two-seater German aeroplane with a revolver. Twelve more were ordered for the Royal Flying Corps and 24 with modifications as Scout C by the Admiralty, followed by further orders in 1915 of 75 and 150 for the R.F.C. In one, Captain Lanoe Hawker, D.S.O., R.F.C., with a carbine fitted to one of the struts, shot down three enemy two-seaters, a culmination of gallantry which brought him the V.C. In 1915 another was flown off the deck of the "Vindex," a converted Isle of Man packet on a Zeppelin patrol; and another took off from under the upper wing of a Porte Flying Boat over Harwich, returning safely to Martlesham.

Subsequently a number of Scout D's, some with 80 h.p. Le Rhone and others with 100 h.p. Mono Gnomes, were ordered and did great service in the Middle East up to as late as 1917. A solitary Scout C, privately owned and flown, survived throughout the 1920's and was finally dumped on a Yorkshire scrap heap in 1930.

In this way a private venture, started almost casually, brought Barnwell into the front line of contemporary designers and provided a most valuable Service aircraft.

(*Right*) The prototype Bristol Scout of 1914, Captain Barnwell's first design for the Bristol Company, with H. Busteed, Test Pilot. (*Above*) The Scout C.



Bristol Scout D (80 h.p. Le Rhone or 100 h.p. Mono Gnome).

The Bristol Scout F Type 21 of 1918 (Mercury engine).

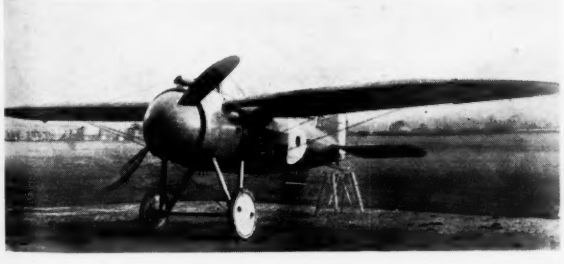
Before, and during the early part of, the First World War it was War Office policy to confine all aeroplane design for the R.F.C. to the Royal Aircraft Factory at Farnborough, Industry being regarded as merely producers to official drawings. By the end of October 1914 the whole of the Filton works found themselves devoted to the production of the BE2C, an admirable flying machine, as might be expected from its designer, Geoffrey (then Captain) de Havilland, with a high degree of inherent stability which war conditions soon showed to be excessive. Pre-occupation with the construction of these aircraft, and of the meagre initial orders for the Scouts, relegated the firm's design and technical staff to a minor role with no creative opportunity, and it is not surprising that Barnwell left to join the R.F.C. He took his ticket in November 1914, got his wings at the C.F.S. in March 1915 (aged 35), and was appointed to No. 12 Squadron in which he did much flying. When, in December 1925, at the age of 45, he applied to join the Institution of Aeronautical Engineers (which made him an Honorary Member), he had logged 360 hours of solo flying as a pilot of 35 different types of aircraft—and that did not include several variants with different types of engine.

During 1915, the appalling losses of the gallant R.F.C. culminating in an outcry in Parliament and violent attacks from the Press, in which C. G. Grey was typically in the van, led to more effective War Office recognition of the essential need to support non-official designs, as had earlier been encouraged by the Admiralty for the R.N.A.S. under its then First Lord, Mr. Winston Churchill. One result was the release from the R.F.C. of Barnwell (now Captain) "on indefinite leave without pay" to rejoin the firm in August 1915 as Chief Designer, Coanda having left on the outbreak of war. He selected as his principal assistant a young graduate in Mechanical Engineering of Bristol University, L. G. Frise, released from a commission in the R.N.A.S.

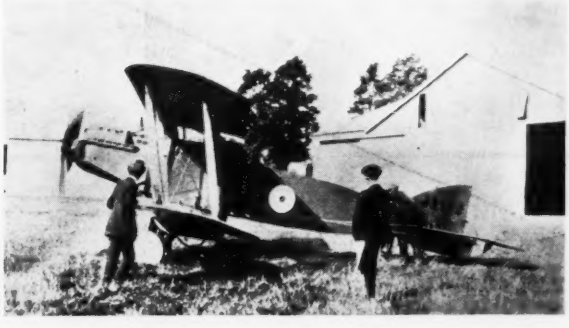
Barnwell with his Service experience addressed himself to two requirements, a single-seater fighter and a two-seater to replace Farnborough types for artillery observation and offensive patrol.

The resultant single-seater fighter Type 10 was a monoplane with a 110 h.p. Clerget in a streamlined fuselage. In September 1916 the first of the five prototypes ordered was flown by F. P. Raynham at a speed of 132 m.p.h. To the disappointment of the firm and others its landing speed of 49 m.p.h. was decreed to be too high for the Western Front and so only another 125 were ordered as Type 11 with 110 h.p. Le Rhone engines, and a Vickers gun mounted on the centre line. The two Squadrons thus armed did good service in Macedonia and Mesopotamia where they took over progressively from the Scout D's in 1917 and 1918.

Barnwell's original two-seater concept Type 9 was based on a 120 h.p. Beardmore engine, the pilot in front with a Lewis gun on the starboard longeron, his eyes level with the upper centre section having a minimum blind spot. The observer gunner was placed immediately behind him on a seat which could be folded out



The Bristol Monoplane M.IC, 1915.



The Bristol Type 12, F2A (190 h.p. Rolls-Royce), of 1916.

The Bristol Fighter, known as the "Brisfit" and one of the most successful aeroplanes of the 1914-18 War.

of the way when he had to stand up to operate his gun. When the prototype came to be built, however, as Type 12 F2A, the more powerful 190 h.p. Rolls-Royce engine had become available and was fitted. The forward-firing synchronised Vickers gun was mounted on the centreline just ahead of the pilot's wind screen and fired through a tunnel in the upper fuel tank, where it was kept comparatively warm by the engine. Barnwell had sent young Frise to an Army machine gun school for a fortnight's course on the Vickers gun to learn its habits. The tail unit was partly below the fuselage which was tapered to a horizontal edge to give the gunner the best possible view and field of fire with his Lewis gun on its Scarff ring mounting. His seat was made to slide as well as tip up and he was near enough to punch the pilot on the shoulder, lacking any other form then of intercommunication. Tail adjustment in flight was provided and also an emergency form of dual control to give the observer gunner a chance to get the aeroplane back if the pilot were knocked out. Parachutes were not then approved for Service use.

The first machine, begun in July 1916, was flying on September 9th, the second with a 150 h.p. Hispano Suisa engine, in October. The prototype had a separate radiator installed vertically on each side of the fuselage, which interfered with the pilot's view and a new circular radiator was therefore designed into the nose.

On the first flight Captain Hooper reported inability to climb above 6,000 ft.; a reading confirmed by Harold Barnwell, who had been called in by his brother. He, however, was sure that he had in fact got higher. On changing the altimeter a height of 10,000 ft. was reached in the first 15 minutes.

So successful were the subsequent official trials that in November 1916 Field Marshal Sir Douglas Haig decreed that two Squadrons should be ready for operations in France for the 1917 spring offensive and 50 aircraft were ordered.

No. 48 Squadron arrived in France in March 1917; and met with shattering disaster. Six fighters led by Captain Leefe Robinson, V.C., in the first offensive reconnaissance of the battle of Arras on 5th April ran into five Albatross led by the fabulous von Richthofen. They attempted to fight back in the then orthodox two-seater manner, the pilots manoeuvring to give their gunners a good field of fire. Four of the six were shot down, two by von Richthofen himself, including Leefe Robinson who was taken prisoner. Six days later four more were attacked by four Albatross, of which two were shot down without loss, but later in the patrol one had to return with a jammed gun and the other three were shot down by four more Albatross. On 16th April six more fighters made a photographic reconnaissance over Douai for nearly half an hour without encountering any enemy aircraft and went on, disregarding a strong westerly wind which caused five to run out of fuel before they could get back to their own lines. The Squadron, unshaken, and increasingly assured of the inherent capabilities and manoeuvrability of the aircraft, then began to operate their fighters as if they were single-seaters, using the front gun for the main attack and the observer's only as additional rear cover. In short, they tore into the enemy with most effective and immediate result, continued by No. 11 Squadron, the second unit to be re-armed with this aircraft.

This striking confirmation of original high hope led to an increase by July in the total quantity on order to

A Bristol Fighter as a trainer.

The Bristol Coupé.

602, with modifications after the first 50 as Type 14 F2B These modifications included continuation of the lower wing to fill in the former gap with a centre section and the re-design of the top longerons to slope downwards from the cockpit to the engine bulkhead to provide a larger fuel tank and ammunition box, and also to improve the pilot's view forward. From the 250th machine onwards the Rolls-Royce Falcon III of 275 h.p. was standardised, with the resultant increase in top speed to 118 m.p.h. at 10,000 ft. and a low level speed of 125 m.p.h.

In July 1917 the War Office decided to re-equip all Fighter Reconnaissance and Corps Reconnaissance Squadrons with this aircraft and production on an unprecedented scale was planned. 4,747 were actually built in this country, nearly one-third of them by other firms than Bristol under a scheme which was virtually the prototype of the Shadow industry of the Second World War.

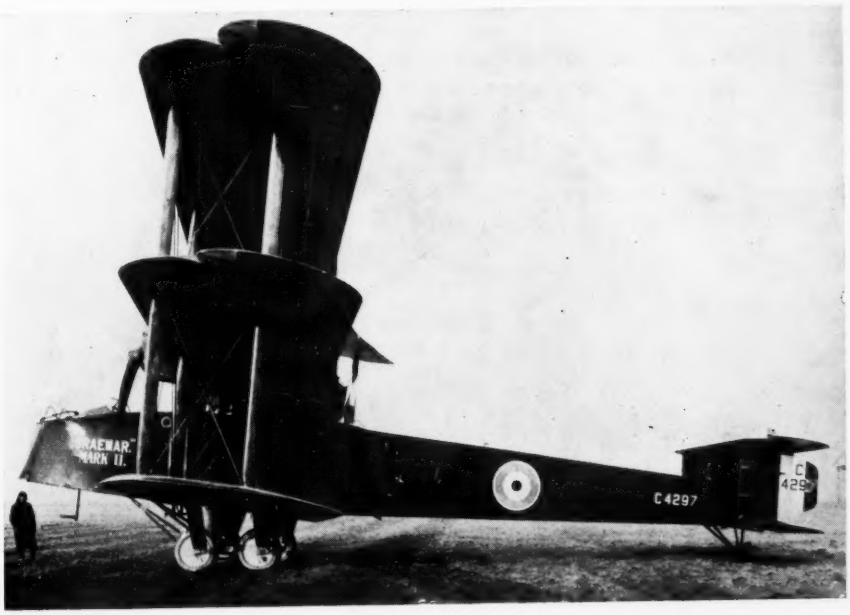
In addition 4,000 were ordered for construction in America, to be installed with the 300 h.p. Hispano Suisa engine, but it was later decided—under high political pressure—to use the newly designed 400 h.p. American Liberty engine, which was far more heavy and bulky than either the Falcon or the Hispano. Barnwell, with the full support of the firm, vigorously but unsuccessfully contested this decision, and the first aircraft flown in America in March 1918 confirmed his apprehension that the machine would be nose heavy and unmanageable. In spite of its brilliant current British record in actual warfare, the F2B was forthwith condemned as dangerous for use by the U.S. Army and only 27 were accepted for experimental flying. In all this there lies a moral!

It may be noted, however, that the Technical Officers of the then Engineering Division at McCook Field, Dayton, Ohio, installed a French 300 h.p. Hispano which, on 18th November 1918, was flown to a height of 29,800 ft. above Dayton, thus gaining unofficially the world altitude record.

In the First World War this aircraft, affectionately known as the "Brisfit," achieved a reputation akin to that gained by the Spitfire and Lancaster in the Second World War. It continued to be of good service in the R.A.F. throughout the 1920's, completing its career about 1928/29 on the N.W. Frontier of India. Indeed, it still lives, as we saw with such delight at the Society's Garden Party in 1953 and the year before—then, I think, piloted by Bill Pegg.

In recognition of his great services Barnwell was invested with the O.B.E. in 1917, to which was added the Air Force Cross. I believe he was the only British designer to be so honoured.

In December 1917 he was elected Fellow of the Royal Aeronautical Society, which he had joined as a Member on 4th March 1914—40 years ago today. In January 1917 he was appointed to a Committee to consider the future of the Society.



The Bristol Braemar, Type 24, which first flew in 1919 and was planned by Captain Barnwell in 1917.

The Bristol Pullman, Type 26, 14-seater civil transport of 1920, and on the right the little Bristol Babe single-seater sporting biplane, first flown in 1919.

The aftermath of the Armistice in 1918 had a far more shattering impact on British aviation, in a drastic financial retrenchment, the unloading of vast war stock on a shrunken market, and industrial upheaval, than was even remotely approached after the second world conflict. The Aircraft Industry was left to fend for itself, with meagre orders and the completion of a few of the prototype aircraft which had been started before the Armistice—that is, those that were not cancelled.

Among the latter may be mentioned briefly: the development of the Scout F Type 21, with a Sunbeam Arab engine and two fixed guns giving a speed of 140 m.p.h. at low altitude, first flown in April 1918, the first Bristol prototype to be flown by Captain Uwins who had been released from the Service to join the firm in October 1918.

The second of this type was fitted with a new air-cooled radial engine, the Mercury, designed by Butler and Fedden for the Cosmos Co., an off-shoot of Brazil Straker, followed by the Badger Type 23, a two-seater fighter with a Cosmos Jupiter completed in February 1919.

Sir Roy Fedden, I know, would wish me to allude to the great help Barnwell gave him in the flight development, cooling, and cowling of the Jupiter and Mercury engines from the start of the Cosmos flight trials; help which continued throughout all the subsequent development of these engines, and the sleeve valve types, within the Bristol Company itself. More than one separate lecture would be needed to do this work proper justice.

In November 1917 Barnwell had schemed a large triplane, the Type 24 Braemar, with internal bomb stowage and a central engine room driving a pair of large propellers through shafting, to bomb Berlin before the end of 1918. Reid took over this project and modified the first prototype to take at first, four Pumas in tandem pairs on the middle wing and later, four Liberties. The aircraft flew in February 1919. The third prototype—ordered and then cancelled by the Air Ministry—was used as a basis for the civil Pullman Type 26, a 14-seater civil transport aircraft flown in 1920.

At the same time Barnwell—with delight— designed in Type 30 a little Babe Mk I single-seater sporting biplane of less than 20 ft. span, which flew in November 1919 with his own 35 h.p. Viale engine.

Type 32, basically the old Scout but with a Cosmos Jupiter, was flown by Uwins in the Aerial Derby in July 1920 and again in 1924.

The old Brisfit took on a new civilian look, Types 28 and 47, as a three-seater Puma-engined tourer, which played a part in an early Continental service. Others were sold abroad. Type 52 (M.F.A.), the Bullfinch, was an ingenious Jupiter-engined single-seat monoplane readily converted to a two-seater biplane by

The Bristol Racer, basically the Bristol Scout, Type 32 (Jupiter), flown by C. F. Uwins in the Aerial Derby in 1920 and 1924.

The single-seat Bullfinch, Type 52 (450 h.p. Jupiter), 1922, which could be either a monoplane or biplane.

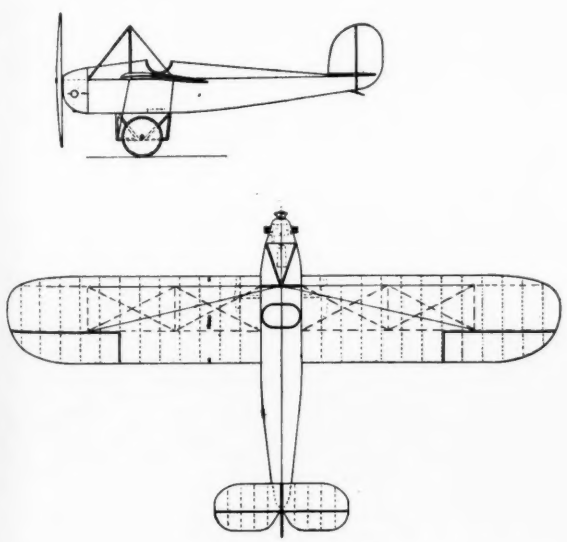
The Bristol Type 62 10-seater transport (Napier Lion), 1921.

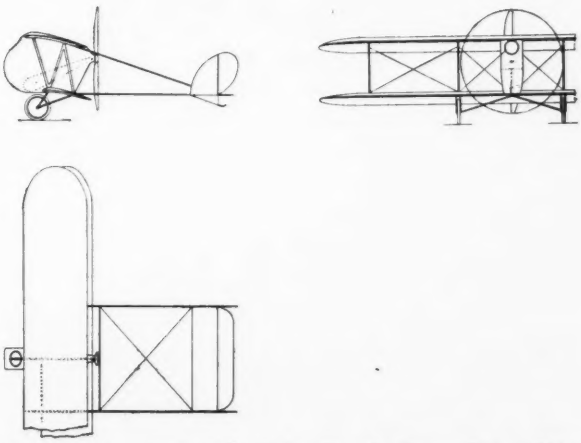
the insertion of a length of fuselage, a structure of steel tubing containing a second cockpit, on the underside of which was mounted a cantilever wing. This was delivered to Martlesham in 1922.

Type 58, a paper scheme for an 8-seater civil cargo or passenger biplane with a Liberty, emerged as Type 62 in May 1921, a 10-seater with a Napier Lion engine and a four wheel undercarriage, with brakes fitted to the rear wheels, which after official trials at Martlesham began experimental services to the Continent from Croydon. The story is that when it took off from Le Bourget, the front wheels were knocked off by a baulk of timber. This had such beneficial results that, after repair, it was flown as a two wheeled type for many hundreds of hours by Handley Page Transport Ltd. on the London/Cologne route. A latter version, Type 75 with a Jupiter engine, became the Bristol Freight Carrier operated by Imperial Airways.

Barnwell's vision and range of thought were illustrated in a paper he read in 1920 before the Cambridge University Aeronautical Society. In it, he outlined: a single-seater monoplane Type 63 with a 5 h.p. engine; Type 64 an aircraft to achieve 380 m.p.h. demanding in his prophetic judgment an engine of not less than 3,000 h.p.; and Type 65 a pusher biplane for limited flight, man-powered to drive a propeller by pedal.

The firm was renamed the Bristol Aeroplane Co. on 31st December 1919. Despite the courage and enterprise displayed by it under the leadership of Sir Henry White Smith (founder of the S.B.A.C.) and Sir Stanley White, Barnwell was clearly unhappy. There was a lack of that complete occupation which he had had throughout the war and of the satisfaction of seeing his design effort materialise in useful production. And so in October 1921 he resigned and took a technical commission as Major in the Royal Australian Air Force to join an experimental section for aircraft design and development in the Commonwealth. But, for reasons not of his making, which some of us well understood at the time, his zest for creative activity did not receive the expected outlet and the lamp of achievement was not re-kindled. And in October 1923, just two years later, he turned up again unheralded at Bristol and was re-appointed Chief Designer, with Frise as his Assistant and the faithful Tinson again his Chief Draughtsman. W. T. Reid, who had been Chief Designer during Barnwell's absence, had recently resigned to join the Canadian Vickers Company.





Two of the types outlined by Captain Barnwell in 1920. (*Left*) Type 63, a single-seater monoplane with a 5 h.p. engine and all-up weight of 310 lb. (with pilot), and (*above*) Type 65, a man-powered pusher biplane with an estimated weight of 180 lb.

The Bristol Brownie Type 91, a low wing monoplane, was built for the Light Aeroplane Competition at Lympne in 1924, with a 24 h.p. Bristol twin-cylinder Cherub engine. Flown by Uwins it won the second prize and no doubt its engine serviceability was much helped by the attention of Freddie Mayer.

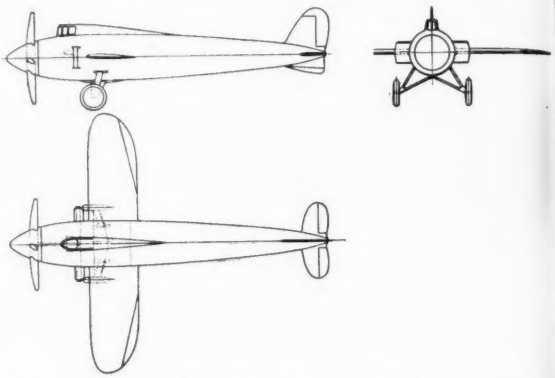
In 1925 a new all-metal Fighter Type 105 was built as a P. V. embodying steel strip construction with equal span wings and carrying the full radio and other standard equipment of that time. In parallel a smaller biplane, Type 107, was built without radio and of minimum weight as an interceptor, for maximum rate of climb. An official order was placed for the smaller 107 but not the larger. But out of these two types emerged the Bristol Jupiter Bulldog for test in January 1928, much of the detail work having been done by Frise and Pollard, who had joined the firm in 1922. Production deliveries began in May 1929 for the R.A.F. Substantial orders were later placed for the Royal Australian Air Force and for Sweden, Latvia, Estonia, Denmark, and Finland. For several years the Bulldog maintained its high Service popularity, although I recall that one of them, while engaged on Service trials which I had instigated on petrol distilled from coal, landed on a winter afternoon in Hyde Park, fortunately without any damage to anyone.

Type 130 based (for once) on an Air Ministry specification, was started in 1933, resulting in the Bombay, a large twin Pegasus-engined high wing monoplane, with a fuselage of Alclad stressed skin, and carrying 24 fully armed infantrymen. This came into production for the R.A.F. in 1935 in some quantity.

In Type 138 Barnwell had planned a single-seater high altitude monoplane; but he failed to obtain any official support for its construction until after the Italians won the World Height Record in 1934, which Uwins wrested back in a Bristol-engined Vickers Vespa with a height of 44,000 ft. Barnwell's 138, first flown in May 1934, with a standard Pegasus for preliminary test work, was fitted later with a special two-stage supercharged Pegasus, and was flown by Swayne and Adams, test pilots at the R.A.E. It took the World's Height Record in the process to 53,937 ft.

In Type 135 Barnwell had drawn out a small twin-engined low wing civil cabin monoplane for six or eight passengers intended for Bristol Aquilla sleeve valve engines. Although it was never built, this was the genesis of Lord Rothermere's interest which led to the construction of Type 142, "Britain First," with two Mercury engines. It flew in the spring of 1935 and on official trial gained a speed of 285 m.p.h., a civil type privately ordered thus giving a speed 30 m.p.h. faster than contemporary R.A.F. fighter performance.

The Bristol Brownie, Type 91 (24 h.p. Bristol Cherub), built for the 1924 Light Aeroplane Competition. Flown by C. F. Uwins it won the second prize.



Captain Barnwell's Type 64 outlined in 1920 for a top speed of 380 m.p.h. with a 3,000 h.p. engine.

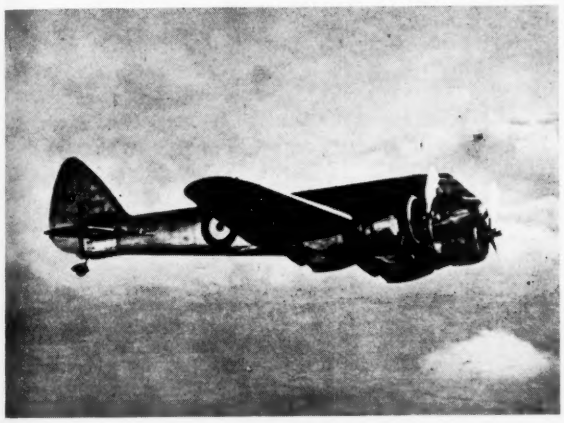
This was followed by a mid-wing version with internal accommodation for 1,000 lb. of bombs with a range of 1,000 miles and a hydraulically-driven dorsal gun turret. This was a private venture later covered by a specially drafted Air Ministry specification 28/35. The resultant Blenheim made its first flight in June 1936, and delivery of production aircraft to the R.A.F. started in March 1937. The Blenheim was destined to be built in large quantities, and to be used as a medium bomber and for other purposes during the greater part of the 1939-45 War.

In fact, the Blenheim, together with Type 152, the Beaufort, derived from it with Taurus engines as a general reconnaissance and torpedo bomber, were to be Barnwell's posthumous contribution to the Second World War.

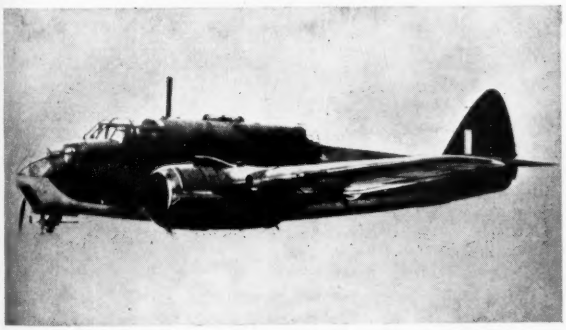
Including design studies which did not come into prototype being, Barnwell in all laid out over 150 types at Bristol within 26 years, a remarkable output, in that he was no figurehead in any one. For many years he designed in detail his own propellers. In 1927 he was awarded the Taylor medal for the best paper presented to the Society during the previous session:—"Some notes on Airscrew Design."

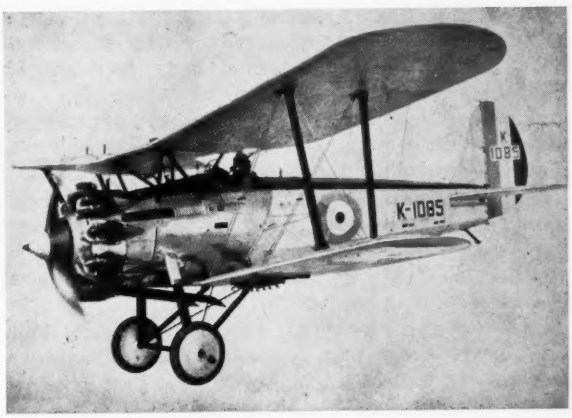
In the middle of 1936, which marks the first stage of the pre-war expansion of the Aircraft Industry, Barnwell was appointed Chief Engineer, the entire drawing office coming under the domain of Frise as his Chief Designer, with four Technical Assistants, A. E. Russell (who had joined the firm from Bristol University in 1925), the late J. M. Radcliffe, H. W. Dunn, and C. W. Tinson. But before that time the younger Frise and Russell and Pollard had been playing an increasing part in the firm's aircraft and development work, thus grafting new thought and advanced outlook on to the old

The Bristol Bombay Type 130 (two Bristol Pegasus) troop carrier, which was started in 1933 and went into quantity production in 1935.

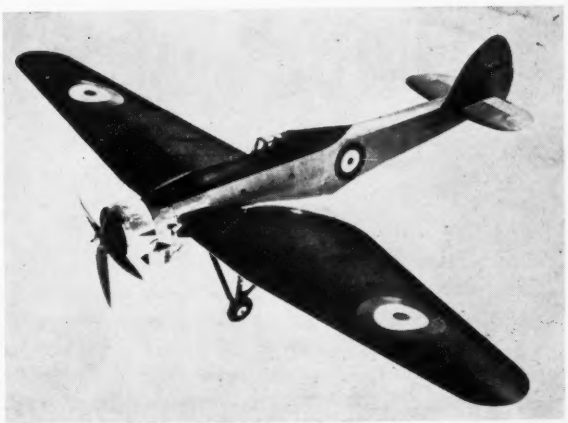


Type 142 (two Bristol Mercurys) "Britain First," of 1935, forerunner of the Blenheim.

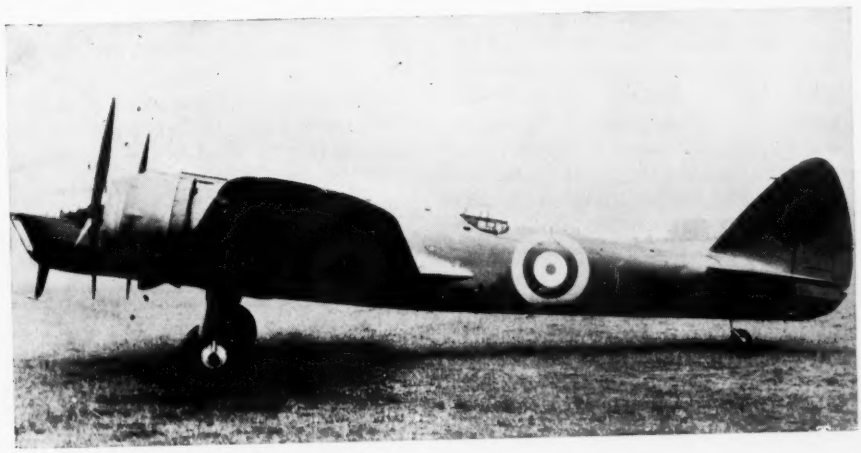




The Bristol Bulldog (450 h.p. Bristol Jupiter), which first flew in 1928 and remained in squadron service with the R.A.F. for a number of years.



The Bristol 138A single-seat high altitude monoplane (Bristol Pegasus), first flown in 1934, which set up a World Height Record of 53,937 ft. in 1937.



(*Right*) The Blenheim IV Fighter-bomber (two Bristol Mercury VIIIs), of 1936, and (*above*) the Bristol Beaufort, Type 152 (two Bristol Taurus), general reconnaissance and torpedo bomber, which first flew in 1938; two famous Bristol types of the 1939-45 War.

tree. This process of grafting was probably not always lacking in argument or clash of opinion. Barnwell, I think, subconsciously no doubt, had never lost his youthful aspiration to build things good to fly in, and to give his fellow mortals the same opportunity for high adventure in the sky. He was an amateur in the original sense of that now suspect word. The demand for ever-increasing wing loadings, the increasing complication of Service equipment to be carried to meet Service exigencies, and even the growing need of civil aviation, must have irked him and it must have taken all the persuasive powers of his younger Assistants for him to accept such things. Conversely, they, I think, have never ceased to be grateful to Barnwell for the experience and wisdom he imparted to them, and the great enrichment of their lives which came from their mutually loyal co-operation.

In 1937, with great reluctance but in his own interest, the firm persuaded Barnwell to cease to fly solo any of the Company's aircraft. He was then 57. But nothing in life would stop him from flying, and in 1938 he designed and built privately at his own expense and helped by a few friends, a little low wing single-seater monoplane of 750 lb. all-up weight, with a 25 h.p. Scott Squirrel two-stroke engine. In it he made a satisfactory preliminary flight in July. Minor adjustments were made to the controls and, in spite of endeavours to dissuade him, he took it up for the second time on 2nd August 1938. As the aircraft began the first turn of a circuit, it stalled, spun in, and he was killed instantly.

So Barnwell died, as he would have wished, in the full harness of his life's work, a consummation which all of us surely crave.

His widow alas was left proudly to mourn the deaths in action of their three sons, one in a Hurricane in Malta, and two in Blenheims in Northern Europe. Thus amply indeed is the name of Barnwell emblazoned in the annals of the conquest of the air, and of service to Country.

I first met Frank early in 1916, and felt always the better for, and inspired by, every meeting throughout the years which followed. He was of slim build and not very tall, with fair hair always kept short, with sparkling light blue eyes which were often quizzical, never flinched, and could freeze. He was meticulously neat and almost fastidious in all his work; in his appearance always soigné, looking usually, I used to think, as if he had just emerged from a Turkish bath and was off to a Point to Point. There was nothing "scruffy" about him. He had a rather staccato, almost hesitant, speech and would sit long silent in conference. Then, sometimes with devastating effect, he would inject a few words which penetrated the heart of the problem. He had a curious habit of shaking the lobe of his ear as he talked, perhaps to conceal mental irritation or as if to stimulate his brain.

I know for myself, 12 years his junior, how ready he always was with patience and courtesy to talk and argue with younger men; and also how patiently he coped with bureaucracy—although I am told that he

The little Barnwell monoplane (25 h.p. Scott Squirrel engine) in which Captain Barnwell was killed on 2nd August 1938.

said once, *sotto voce*: "We struggle against a sea of bloody fools." He could be stubborn indeed, but there may be some who will say that, if he had failed to get acceptance of his argument or if he had been prevented from doing what he was convinced was right, he would break off the battle and retire to his ivory tower instead of hurling himself in desperation against the barricades. But if that were true there was no sign of lack of moral courage and self respect. There was nothing sullen in his make-up, and if decisions were made contrary to his judgment he was ready always to do his best to mitigate the ill effects he had foreseen.

How would Barnwell have stood up to the aeronautical problems of today, with all their enormous increase in complexity and the need for intensive specialisation in so many contributory fields? It would be as idle to speculate on that as it would on what Lord Palmerston might have done with Hitler, or W. G. Grace with the bowling of Rhamadin. Of one thing I feel sure, Barnwell would still be an outstanding leader of the team. The more complicated the modern symphony, the more gifted the individualists, the greater the need for a *maestro*, a virtuoso, to conduct the orchestra. The modern aeroplane, like every other enterprise in modern engineering, depends for its success on the wise selection of the best compromise—plus something we call genius born of art, and wisdom. There is a recognisable cut about an aeroplane which identifies the man responsible for its design.

In these days when the advance of scientific discovery is ever accelerating; in these days when man is able to synthesise nearly all that "makes things tick;" in these days of the frightening potentiality of digital computors and all the other electronic black magic; there is, I think, a risk. There is a real danger that the attributes of the human heart and brain, the qualities of experience and judgment, the power to select and direct, will incline to become stultified and atrophied. Man, I fear, is tending to become too content to be the servant of apparatus, mesmerised by mechanism; and his ultimate goal, his New Jerusalem—universal push button control; when any clumsy thumb will do to push a button with. Instead of a "bow of burning gold" and the "arrows of desire," man is calling increasingly for an armchair made of plastic and a pillow of unstriving ease.

Not so Frank Barnwell. For him the driving purpose and the limitless horizon; for him the opening

vista of new thought and the constant stimulus of individual effort. He was a man to follow and to emulate.

In this " brief traffic of an hour " I have been able to give only a faint impression of a great man and his achievements, without I hope too many inaccuracies of detail. Future Barnwell Memorial Lectures will no doubt be highly technical, and devoted increasingly to aspects of our profession dealing with theories and data of which Barnwell, who died nearly 16 years ago, could have had no perception and yet which may well have had their genesis in some of the work he did. If the example of this great Englishman, his character, unswerving technical honesty, high ideals and devotion to duty—if these are not forgotten, British aviation will have no cause to fear the future.

FURTHER TRIBUTES

THE PRESIDENT: You have already shown by your applause how deeply you have been moved by Major Bulman's tribute to Captain Barnwell.

This is not the kind of lecture on which a discussion is either usual or appropriate. But in view of its personal character, and in view of the fact that there are present one of two people who knew Frank Barnwell perhaps even better than our lecturer, I will ask if three of them will say a few words in the way of personal reminiscences, which I feel will add point to the lecture we have heard. They are Mr. C. F. Uwins, Mr. C. W. Tinson and Mr. L. G. Frise.

C. F. UWINS: Normally I dislike speaking at lectures, but on being invited to speak this evening I accepted with great pleasure because the lecture we have heard is one on which there can be no controversy and I have two particular tributes I wish to pay.

The first tribute is to our lecturer, Major Bulman, for a magnificent, sympathetic and nostalgic address, something more than a lecture, almost an oration. To those of us who have been associated with the work of Captain Barnwell and that of our Company, the lecture has brought home what a task has been undertaken over the past thirty years. I have been most moved, as I am sure many of my colleagues have been, by the way in which Major Bulman paid his tribute to Captain Barnwell; and I would like to thank Major Bulman personally for doing it so sympathetically.

I always think of Captain Barnwell under two particular headings, (1) as a technician and as Chief Designer with whom I worked for a matter of 21 years, and (2) as a man. Of his aircraft, with possibly one or two exceptions—I am not quite certain—I think I flew every machine, including the Scout D. On reflection, the one I enjoyed most was his second machine, the single-seater monoplane with a Le Rhone engine; it was almost the finest aeroplane to fly, from the pilot's point of view. But of the aeroplanes brought to fruition by the Company—the Bristol Fighter, the Bulldog, the Blenheim—all had an integrity about them which reflected that of the Chief Designer, Captain Barnwell; that integrity communicated itself to all the pilots and crews of the Royal Air Force who flew those machines in service, and in consequence it gave them a greater confidence in their machines. I well remember, even before I joined the Company, that in the Royal Flying Corps it was always said that whatever one did with the Bristol Fighter it would never break up in the air, and that is one of the great reasons why it was so magnificently flown by the R.F.C. pilots and why it had such a splendid record in the First World War.

Having regard to the aeronautical knowledge we have accumulated in these days, I realise how sketchy was the knowledge available 30 years ago. Captain Barnwell had a keen appreciation of exactly what would happen to his aircraft; that is a source of profound consolation to a young test pilot whose happy conduct of his affairs largely depends on the correct prognostications of his Chief Designer. As a case in point, I remember the aeroplane of which you have seen a photograph this evening, the Bullfinch, with the high cantilever wing. With the version with the cantilever wing below the fuselage, I recall that people at the Royal Aircraft Establishment, or whatever it was called in those days, predicted that the flow over the tail would be such that a change of incidence should be provided, under the control of the pilot, of at least 20°. Captain Barnwell was terribly angry, and said that 2° was probably the right figure! I think there was practically no change of trim. He was so right and so many others were so far wrong.

He loved designing propellers; it seemed that he designed one every day, and he had a family of curves whereby he could design one in about five minutes. When our Engine Division had one of our engines—made by the French Licensees Gnome et Le Rhone—installed in a German aircraft for an attack on the then World's Height Record, repeated attempts met with repeated failure. Information was provided to Captain Barnwell and he designed a propeller; the machine with that engine and propeller forthwith achieved a new World Height Record. That indicates the calibre of the man and his knowledge.

When I think of him as a man I do so with the warmest regard and respect. I was very close to him and I knew him uncommonly well. His manner conveyed the impression of age beyond his years and he was affectionately known among us as "Daddy Barnwell;" I think he will always be remembered by that name. But although he appeared to be old in years he was very young at heart and in many respects he never grew up. In business matters he was a child, and he would fall for any hard luck story. Of his courage I speak with the highest regard. He would fly anything, knowing no fear. He

was warm-hearted and gentle and incapable of an unworthy thought.

It is fitting that such a man should be commemorated by an annual lecture and I am gratified that this most active Branch of the Society, of which Captain Barnwell was the first President, now has this Barnwell Memorial Lecture placed in its charge.

C. W. TINSON: When I first came here to Bristol at the end of 1911, from the Universal Aviation Company of London, I was immediately conducted to the cottage in which Barnwell, who had joined the firm a few weeks earlier, had been installed and was preparing to go to work on the development of Lieutenant Burnie's ideas and projects.

I count myself fortunate, in the first place to have been recommended from London to the Bristol Company, and, secondly, to have been chosen and appointed as Barnwell's assistant in this novel field of research.

We worked together in great contentment and perfect harmony for nearly three years, for the most part on our own, but were joined for a time by Harry Busteed—Air Commodore Busteed, who is here with us to-night—who occupied the back room or kitchen and lustily sang a ditty called "Back to my Home" most of the time. You may gather that it was a happy association—and it was.

We worked on the two Burnie Boats, on the Gordon England biplanes, and finally on Barnwell's first free-lance biplane S.N.183, sometimes known as "Busteed's Machine" and later as the Bristol Scout when it was in production early in the 1914-18 War, the success of which paved his way to fame.

As you know, Barnwell then decided to leave to join the Royal Flying Corps and I left also, having spent the happiest years of my life working with him.

He was ten years older than I, and was at that time at an age when some are tempted to throw their weight about a bit, particularly where a junior is concerned; to air and emphasise their superior knowledge and wider experience. He was never like that. He was extremely modest and unassuming, very genuine and sincere, and entirely lacked any form of arrogance. While he was not a man to suffer fools gladly, he was ever polite and would never say or do anything unkind to rebuff or humiliate a person. He was kind-hearted, good-natured, scrupulously fair and absolutely straight.

He was a grand man—and a great one, for we must not forget that the industrial prosperity of a City is largely bound up with the deeds and accomplishments of men among whom Barnwell must be numbered.

When I returned to Bristol in 1923 and was again on his staff, he had won fame and renown as an aeroplane designer, but was unchanged by success and remained just as modest, just as unassuming and just as likeable to the end.

Barnwell was essentially a draughtsman—and a first-class one at that—with a sound technical training and engineering background. In the early days, aviation attracted to itself its fair share of enthusiasts whose principal qualifications were a fertile imagination and an inventive turn of mind, sometimes with an inadequate engineering foundation on which to build. Barnwell was not numbered among them. He had an extremely logical approach and would ponder a problem deeply, surveying it from every angle, before making up his mind. Then, he would turn to his drawing board and register the results of his deliberations.

He was a draughtsman of the old school, of the days before mass production, and was extremely neat and methodical in everything he did. I feel that his early environment must have been similar to where I was apprenticed, where designs were prepared on cartridge paper in Indian ink, with centre lines drawn in red and dimensions lines in blue, the complete job being more or less a work of art. He had a very high standard as a draughtsman and maintained it throughout his life. He would take as much trouble over the smallest detail as on a major design.

Because he was so painstaking and thorough, he set an example to those around him which many consciously or unconsciously followed, and among those who were on his staff it is not difficult to identify his disciples for that reason.

It is hardly necessary to add that everyone was intensely loyal to him, and to their loyalty was joined a sense of personal affection and regard which reacted beneficially, both to the individual and collectively as a firm.

Captain Barnwell was indeed beloved and respected as a man and admired as a designer by all whose good fortune it was to work with him.

L. G. FRISE (Technical Director and Chief Engineer, Percival Aircraft Ltd. *Fellow*): I knew Captain Barnwell for 22 years. The impact of such a personality over that time leaves such an impression on those who experienced it that we can never record all that he meant to us without deep emotion. Indeed, for that reason I did try to avoid talking about him, for it recalls the shock of his sudden death and all that it meant to us, because if ever there was a man who was the Captain of a happy ship it was Captain Barnwell. He had such affection from us; I would have doubted that the passing of any one individual could have affected so many so deeply as it did.

But these thing happen. We want to recall the fun we used to have with him, because he was great fun. When I first met him I had been sent down to the Bristol Aeroplane Company from the R.N.A.S. to help in the design of a new aircraft, as I had only just left the University. I was interviewed by Captain Barnwell; he seemed to think I could help, and so I was sent over to the big house where the financiers dwelt. When I came back he asked "How much are they going to give you?" I replied "Thirty-five bob a week." He pulled the lobe of his ear and said "Good God! You have first class Honours, haven't you?" I said "Yes." He said "A ruddy tram driver gets more than that." I replied "Yes, but I took mechanical, not electrical."

However, we got down to it, and I found that the sums I had to do were concerned with what became the Bristol Fighter. As Captain Uwins has mentioned, we

had a very good run of luck and there were no structural failures, not because my arithmetic was good, but because Captain Barnwell told me the right sums to do. That, I think, is still the most important feature of all these calculations; I know these things are more complicated today, but you can waste a lot of time by doing the wrong sums. He had a flair for aircraft design, although our knowledge was so ridiculously small then that one sometimes wonders that the things ever worked at all.

I was for so long with Captain Barnwell that we passed through all the stages of what I believe were called improvements—at any rate, complications—in aircraft, and he rather got a name, which I think he more or less encouraged, with his tongue in his cheek, for being conservative in technical matters as well. As we saw the advent of variable pitch airscrews, landing flaps, retracting undercarriages, cantilever wings, etc., he would sometimes fight to the last ditch to prove that they were not necessary, and then promptly get into a huddle with us and we would proceed to do just those things. Some people thought, by the argument he would put forward, that he meant it, but I do not think he did. He loved the argument and approved the final products.

There is no question of his personal courage, which was extraordinary; I suppose that was the reason we lost him in the end. I remember that in the early days I did a lot of flight recording in the back seat of the Bristol Fighter, sitting on sandbags and with my feet in two holes, gradually freezing to death from the feet upwards. On one occasion the front cross wires broke off, and every time we turned the wings swayed to one side or the other. When I pointed it out he laughed and I said that I did not think it was funny. He said "There is no load in them." and I said I knew, I did the sums, but I would still like to go home if he didn't mind, because I was freezing to death as well.

The reasons why people became so fond of him and would work all hours of the day and night were that he set an example and did the same and was such a good sport. I remember when we lived in an annexe behind Rodney Hall, approached by a wooden corridor, we would enliven the hours with fun and games, and one day tied a shock absorber on one of the beams and took turns in swinging from one end of the building to the other. On one occasion, when one man was swinging full out, somebody said "Here's the old man coming down the corridor," and we all scuttled back to our boards, leaving the victim suspended. The old man looked as if he did not notice anything; although as he talked at a board the chap kept swinging violently past his head; he pulled both ears and went back into his office. We then released the man swinging who said "Why didn't some so-and-so lift me down?" Later Captain Barnwell's only reference to it was to say "What sort of shock absorber did that chap have? It must have been ruddy strong."

On one occasion when he came back from Australia there was a big biplane waiting to be put right. It had different wing chord top and bottom, different sweepback top and bottom; the poor bloke who had calculated the lengths of the interplane struts had made a mistake and was trying to put it right before the old man found out. Captain Barnwell came limping back into the office, having seen the struts in the shops and heard comments by the foreman, and he said the interplane strut was wrong. When it was suggested that the end fitting could be adjusted, Captain Barnwell said "It isn't that wrong, it's b——y wrong; it's a foot short!"

But however he treated the fellows, it was all in good fun. I do not think you could ever find a more loyal team and a team which did stay put. I am sure that everyone in that team has to thank Captain Barnwell for giving them the right approach to everything, with both feet well on the ground. Bearing in mind that up to 1938, when he passed on, our knowledge was not all that complete, the fact that he instructed so many chaps who have since made good in the more complicated world that we know today is a great tribute to him as the father of us all.

THE PRESIDENT: I think you will agree that I was right to exercise my persuasiveness on Mr. Frise, because it shows how the most retiring can contribute so much on an occasion like this.

I have many memories of Captain Barnwell. A passion we both shared was that we liked flying for its own sake and we wanted to keep on flying as long as we could. I must confess, although it led to his death, that I think he was absolutely right.

I am proud to say that I worked at the Royal Aircraft Establishment. On one occasion—I do not remember exactly in what year it was, but it must have been about 1917—I was on the aerodrome at Farnborough; it was getting rather late and I was rather cross because the aeroplane I was going to fly became unserviceable, which was not unusual in those days. There came a Bristol Fighter, and a very irate young man said "I came to leave this thing here, and would somebody fly with me back to Brooklands before dark?" It had an R.A.F.4 engine in it. I rather stupidly said I would go, and I got into the back. He flew me to Brooklands. On landing, he did not stop the engine, but he went to the other end of the aerodrome, hopped out and said "In you go and off you go." I did. I landed by the lights of the "factory" as we called it, for there was nothing much else to show me exactly where it was. But in those days we knew, as he knew, there were some good aeroplanes.

He was indeed a happy man. I think all of us who had the privilege of knowing him must feel that it was in what he did here that his great happiness lay, as I am sure you who did not know him will have gathered from those of his contemporaries who have spoken here.

I would like to propose a formal vote of thanks to Major Bulman, and I ask you all to show your appreciation of his "oration," as Mr. Uwins called it.

The Estimation of Fatigue Damage in Aircraft Wing Structures

by

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SUMMARY:—A simple cumulative fatigue damage law is used to estimate the endurance of a number of typical structural components when subjected to the alternating wing loads encountered by an aircraft in flight. A method is developed to study the fatigue damage in terms of infinitesimally small intervals of alternating load and the corresponding fatigue strength. Recent gust data are used in conjunction with the complete fatigue strength diagrams of 13 simple components. The fatigue damage is studied for the ranges of altitude (*i*) up to 12,000 ft., and (*ii*) above 30,000 ft. In each case it is found that the damage rate curve, which gives the intensity of fatigue damage at any gust velocity, takes a characteristic form, with a well-defined range of gust velocities giving the greatest fatigue damage. This leads to a simplification of the whole problem, and it is found, finally, that the life of the structure is governed—in so far as the cumulative law is correct—almost entirely by that gust velocity, which, when applied alone to the structure, gives an endurance of about two million cycles. The results of the survey are compared with the design criteria suggested by Walker[5] and the lives estimated by Williams.[6]

1. Introduction

A common feature of aircraft wing design in this country is the use of a relatively small number of main joints to withstand the steady and alternating stresses induced during flight. When the designer turns to a high-strength light-alloy material from which to fabricate the joint, he is confronted immediately with a complex fatigue problem. The problem arises because the more common high-strength light-alloy materials give no indication of a fatigue endurance limit, and materials of this sort are prone to fatigue failure over the whole range of alternating stresses. If the bending actions on the aircraft wing were resisted by a large number of wing joints—each one carrying a relatively small fraction of the total load transmitted—then the occasional fatigue failure of a joint would not be regarded as affecting seriously the ultimate strength of the wing; on the other hand, the fatigue failure of one of a few wing joints may be catastrophic, and for this reason the designer must have some indication of the probable life of the wing joints under flying conditions.

The fatigue problem can be defined as the estimation of the life of a structure, or single component, when it is subjected to alternating loads of varying intensities and frequencies, superimposed on the steady loads of level flight. The problem can be solved only approximately at this stage for two important reasons: firstly, our knowledge of fatigue strength, and the mechanism of fatigue damage, is seriously limited at this stage, while, secondly, we have only a general or overall picture of the alternating loads which are encountered in flight. We are therefore restricted in our discussion to the study of an idealised problem relating to the aircraft of average strength subjected to alternating loads, which, on the average, follow certain well-established laws.

The estimation of fatigue damage in aircraft structures is based essentially on the study of the effects of a set of alternating loads of known intensity acting on a structure of known fatigue strength. For this reason it is important to have available as much information as possible on the frequency of alternating loads encountered in flight and on the fatigue strength of typical wing joints; these two aspects of the problem are discussed more fully.

Notation

U_e	equivalent vertical gust velocity (ft./sec.) (*see* Ref. 2)
m	number of miles flown for U_e to be equalled or exceeded in intensity
r	gust velocity ratio ($=U_e/50$)
r_c	characteristic gust velocity ratio
m_0	number of miles flown for zero gust velocity ratio to be equalled or exceeded
m_1	number of miles flown for unit gust velocity ratio to be equalled or exceeded
$E(r)$	number of times the gust velocity ratio r is equalled or exceeded in each mile of flight
V_1	equivalent air speed (ft./sec.)
ρ_0	standard air density at sea level (slugs/ft.^{3})
k	alleviation factor
a	slope of the lift curve
w	additional wing loading intensity caused by a vertical gust of velocity U_e ft./sec.; defined by equation (2); (lb./ft.^{2})

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w_L intensity of wing loading in level flight
W intensity of wing loading for static failure of the wing joint
$N(r)$ endurance in cycles of the wing joint to repeated applications of the gust cycle $\pm r$, superimposed on the level flight loads
Δ total fatigue damage to the wing joint in one mile of flight, defined by equation (5)
S "fatigue stress factor" used in Ref. 6.

2. Loads Encountered in Flight

In the level flight of an aircraft in still air the main components of the wing structure are subjected to steady loads corresponding to the total weight of the aircraft. Additional loads arise in these components when the aircraft encounters vertical gusts, when manoeuvres are made, and during landing and take-off. Fatigue damage in the wing structure is caused largely by those low velocity gusts which occur relatively frequently in flight.

An extensive study of low velocity gust loads in civil aircraft has been made by Taylor[1]. In this work a sensitive counting accelerometer has been used to find the frequency of occurrence of accelerations of different intensities. In order to eliminate the idiosyncracies of the aircraft type from the acceleration statistics, the records are given in terms of equivalent vertical gust velocities, which are derived from the accelerometer readings by an orthodox formula[2].

In applying such gust data to fatigue problems it is implicitly assumed that the additional stresses in the structure are directly proportional to the incremental accelerations experienced by the aircraft. This assumption has been justified by extensive research[1]. It has been suggested by Jackson and Grover[3] that for gusts of any intensity there are an equal number of up and down gusts over long distances of flight, and, moreover, that it is more probable that an up gust is followed by a down gust than that two up or down gusts occur simultaneously. In applying gust data to the estimation of fatigue damage it is convenient to assume that an up gust is always followed by a down gust of equal intensity. When the problem is discussed in this way the gusts are treated in terms of complete cycles of alternating load which are superimposed on the steady loads in the wing joints due to level flight.

The frequency of gust loading on an aircraft is largely dependent upon the altitude at which the aircraft is flying; up to 12,000 ft. a gust velocity of 10 ft./sec. (E.A.S.) is equalled or exceeded once, on the average, in every 10 miles of flight[1]; for flying above 30,000 ft., however, the same equivalent gust velocity is equalled or exceeded once in about 2,000 miles[1]. At the lower altitude, gusts of 10 ft./sec. or greater are therefore two hundred times more frequent than at the higher altitude. It is important then to relate fatigue damage to the altitude of flying; over the same distances of flight, and for the same aircraft, fatigue damage at lower altitudes is considerably greater than at higher altitudes.

If m miles are flown for the equivalent gust velocity U_e ft./sec. to be equalled or exceeded, then according to the statistics collected by Taylor[1] there is a linear relation between $\log_{10} m$ and U_e, the constants in the relation depending only upon the altitude of flying. Taylor's gust data refer to the sum of up and down gusts of the same magnitude; in the fatigue problem we are more concerned with complete cycles of loading, and considering the gust velocity cycle $\pm U_e$ ft./sec., we find that this cycle is equalled or exceeded in intensity E times in each mile of flight, where E is given most conveniently in the form

$$E=\frac{1}{2m_0}\left[\frac{m_1}{m_0}\right]^{-r}, \qquad . \quad . \quad (1)$$

in which $r=[U_e \text{ ft./sec.}]/[50 \text{ ft./sec.}]=$ the gust velocity ratio,
$m_0=$ number of miles flown for the zero gust velocity ratio to be equalled or exceeded, and
$m_1=$ number of miles flown for unit gust velocity ratio to be equalled or exceeded,

for the particular altitude under consideration. In this form E is dependent upon the non-dimensional gust velocity parameter defined by r. For flying up to 12,000 ft. the relevant values of m_0 and m_1 are given by

$$\log_{10} m_0=\bar{1}\cdot 20, \quad \log_{10} m_1=8\cdot 20,$$

while for flying above 30,000 ft.

$$\log_{10} m_0=0\cdot 85, \quad \log_{10} m_1=13\cdot 42.$$

3. Relation Between the Gust Velocities and the Alternating Loads in the Wing Joints

The additional wing loading intensity, w, caused by a vertical gust of velocity U_e ft./sec. is

$$w=[\tfrac{1}{2}\rho_0 k a V_i]U_e, \qquad . \quad . \quad . \quad (2)$$

in which $V_i=$ equivalent air speed (ft./sec.),
$\rho_0=$ standard air density at sea level (slugs/ft.^{3}),
$k=$ alleviation factor $=0\cdot 8-1\cdot 6\,w^{-\frac{1}{4}}$, and
$a=$ slope of the lift curve.

In British design the main joints of the wing are effectively designed to withstand a certain multiple of the loads induced by the combined effects of the level flight loads and a 50 ft./sec. gust. Suppose W is the wing loading intensity for static failure of the joint, and that w_L is the intensity of wing loading for level flight of the aircraft. From expression (2) we can calculate the wing loading intensity, w_{50}, for a 50 ft./sec. gust. Then the joint must be such that

$$W \geqslant 1\cdot 5\,[w_L+w_{50}].$$

If we consider the minimum requirement in this condition, we have that

$$w_{50}=\frac{W}{1\cdot 5}-w_L.$$

In terms of W and w_L, equation (2) then becomes

$$w=\left[\frac{W}{1\cdot 5}-w_L\right]\frac{U_e}{50}.$$

Now $U_e/50$ is the gust velocity ratio r; so that

$$r = \frac{w}{W}\left[\frac{1}{\frac{1}{1{\cdot}5} - \frac{w_L}{W}}\right] \quad . \quad . \quad . \quad (3)$$

Equation (3) gives the gust velocity ratio in terms of (w/W) and (w_L/W); these are the parameters normally used for the presentation of fatigue strength data. The parameter (w/W) is the ratio of alternating load to the ultimate static strength of the wing, while (w_L/W) is the ratio of mean load to the ultimate static strength. Equation (3) is dependent entirely upon the design requirement being only just satisfied. If the design requirement is more than satisfied then we must return again to equation (1) and determine the extent to which the joint is over-designed.

We may use equation (3) to express all data on fatigue strength in terms of the endurance N, in cycles, to the alternating load effects of a gust velocity ratio r; in this way the fatigue strength is the endurance to the repeated application of the loading cycle caused by two gusts, one up and one down, of velocity U_e. This loading cycle is superimposed on the mean loads corresponding to the level flight of the aircraft.

4. The Fatigue "Damage"

The estimation of fatigue damage consists in relating the frequency of gust velocities to the strength of the wing joint under repeated loads. If the gust "cycle" $\pm r$ is equalled or exceeded $E(r)$ times in each mile of flight, then in the interval of gusts from $\pm r$ to $\pm(r+\delta r)$ there are

$$E(r) - E(r+\delta r) = -\delta E$$

complete gust cycles in each mile of flight. We shall assume that tests have shown that the wing component can withstand N repetitions of the gust cycle $\pm r$—superimposed on the appropriate mean load—before fatigue failure occurs. We shall assume further that a single repetition of the gust cycle damages the component to the extent of $1/N$, and that the damage due to n repetitions is n/N, and so on[4]. In this case the fractional damage due to $-\delta E$ repetitions of the gust cycle $\pm r$ is

$$\delta\Delta = -\delta E(r)/N(r),$$

provided that the increment δr is sufficiently small. Then in the limit, as $\delta r \longrightarrow 0$,

$$\Delta' = \frac{d\Delta}{dr} = -\frac{1}{N}\frac{dE}{dr} \quad . \quad . \quad . \quad (4)$$

The total damage due to all gust cycles is

$$\Delta = -\int_0^\infty \frac{1}{N}\frac{dE}{dr}\,dr, \quad . \quad . \quad (5)$$

in each mile of flight. The integration is extended over the whole range of gust cycles, from 0 to ∞. On this basis the total fatigue damage in M miles of flight is $M\Delta$. A fatigue failure of the wing component is assumed to occur when $M\Delta$ becomes equal to unity. The life of the component in miles of flight is $1/\Delta$.

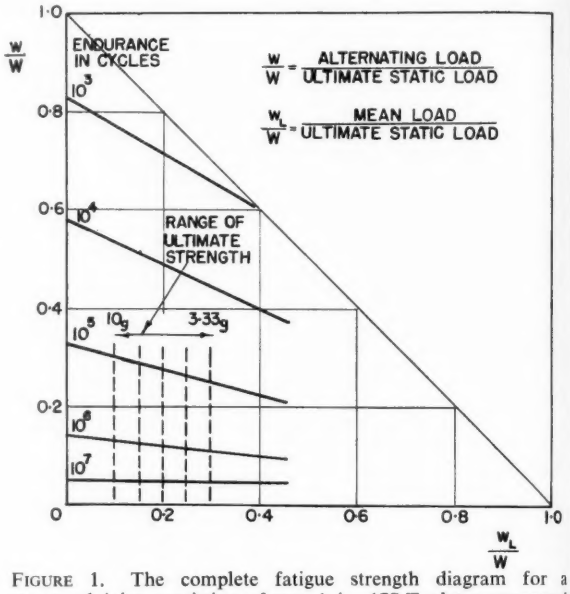

FIGURE 1. The complete fatigue strength diagram for a structural joint consisting of two $\frac{1}{4}$ in. 17S-T plates connected by a single row of hot-driven $\frac{5}{8}$ in. 17S-T rivets. (*See* component No. 2, Table II.)

The relation for $(d\Delta/dr)$ is an important one because it defines the fractional damage per gust velocity ratio at any gust velocity intensity. The function $(d\Delta/dr)$ is then essentially a damage rate with respect to the gust velocity ratios; it is continuous over the whole range of values of r and attains a maximum value at the gust velocity causing the greatest fatigue damage.

The concept of a continuous damage rate is especially useful when applied to Taylor's gust data. This is so because on substituting for $E(r)$ in equation (5) the value given by equation (1), the total damage becomes

$$\Delta = \log_e(m_1/m_0)\int_0^\infty \frac{E(r)}{N(r)}\,dr, \quad . \quad (6)$$

in each mile of flight. In this way the derivative of E in equation (5) is eliminated; the integral in equation (6) is easily derived by comparing corresponding values of E and N.

5. Fatigue Damage in Typical Structural Components

The analysis of the preceding paragraphs has been used to study fatigue damage in a number of structural components with well-defined fatigue strength characteristics. The fatigue strength of a component may be deduced only from an extensive range of alternating load tests. In the United Kingdom published experimental work has dealt with only a few particular loading conditions corresponding to the loads estimated to occur during the life of the aircraft; with such a limited knowledge of fatigue strength it is difficult to study the problem in a general way. American writers, on the other hand, realising perhaps the need for a wider knowledge of the fatigue strength of structures, have

studied the behaviour of a number of typical structural components over a relatively wide range of loading conditions.

A certain simple riveted joint in a light alloy material has the fatigue strength characteristics shown in Fig. 1. The strength is presented in terms of endurance at mean tension loads and alternating loads of varying intensities. Obviously an experimental survey of the type given in Fig. 1 is not essential when the designer is concerned with the strength at one mean load only. The region of the diagram chosen for particular study is that corresponding to ultimate static strength factors ranging from $3{\cdot}33g$ to $10g$. Load-endurance curves are first derived for the particular values $w_L/W = 0{\cdot}10$, $0{\cdot}15$, $0{\cdot}20$, $0{\cdot}25$ and $0{\cdot}30$. These curves are given in Fig. 2 in terms of r, which is defined by

$$r = \frac{w}{W}\left[\frac{1}{\frac{1}{1{\cdot}5} - \frac{w_L}{W}}\right].$$

The curves give the fatigue strengths of five structural components, all of the same type, carrying mean loads ranging from 10 per cent. of the static ultimate to 30 per cent. of the static ultimate.

The next step in the analysis consists in estimating the fatigue damage to the five structures represented in Fig. 2. At the same altitude each of the five components will be subjected to the same gust loads. For flying up to 12,000 ft. the fatigue damage rate curves take the forms shown in Fig. 3. These curves are characteristic of all the damage rate diagrams studied. In each curve there is a pronounced peak at a certain gust velocity ratio. To the low velocity side of this peak the damage rates fall off rapidly, and there is a range of low gust velocities for which there is no fatigue damage at all. To the high velocity side of this peak the damage rates fall off less rapidly. It is interesting to note that the gust velocity ratio giving the highest damage rate is not

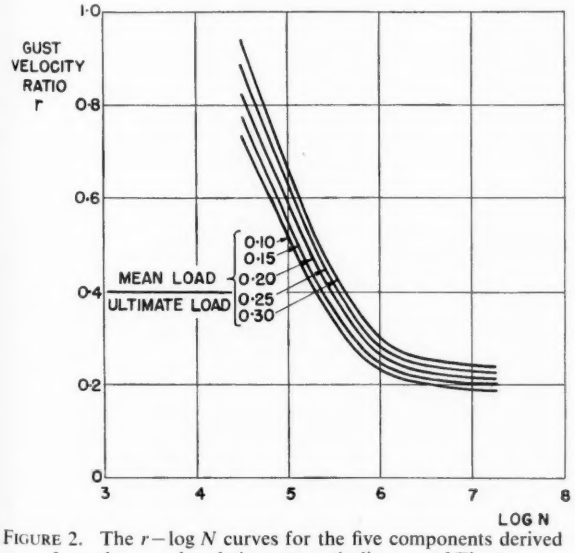

FIGURE 2. The $r-\log N$ curves for the five components derived from the complete fatigue strength diagram of Fig. 1.

TABLE I

FATIGUE DAMAGE TO THE FIVE STRUCTURES DESIGNED ON THE INFORMATION CONTAINED IN THE COMPLETE FATIGUE STRENGTH DIAGRAM OF COMPONENT NO. 2 (TABLE II)

(Mean load) ÷ (Ultimate static load)	$\triangle$ = *Area under damage rate curve*	*Log* $\triangle$	*Log (life in miles)* $= \log 1/\triangle$
0·10	$7{\cdot}19 \times 10^{-8}$	$\bar{8}{\cdot}86$	7·14
0·15	$5{\cdot}02 \times 10^{-8}$	$\bar{8}{\cdot}70$	7·30
0·20	$3{\cdot}18 \times 10^{-8}$	$\bar{8}{\cdot}59$	7·41
0·25	$2{\cdot}54 \times 10^{-8}$	$\bar{8}{\cdot}40$	7·60
0·30	$1{\cdot}78 \times 10^{-8}$	$\bar{8}{\cdot}25$	7·75

the same for each structure. For flying above 30,000 ft. the damage rate curves take similar forms as shown in Fig. 4.

The final step in the analysis consists in evaluating the areas under the damage rate curves; the areas correspond to the total fatigue damage in each mile of flight (Table I).

Using this method of analysis the complete fatigue strength diagrams—of the type shown in Fig. 1—have been studied for thirteen different structural components. Brief details of the components, with the sources of reference, are given in Table II. Five structures with ultimate strength factors varying from $3{\cdot}33g$ to $10g$ have been studied for each of the thirteen complete fatigue strength diagrams. This has involved the determination of the fatigue damage rate curves for 65 structural components operating

(*i*) up to an altitude of 12,000 ft., and
(*ii*) above an altitude of 30,000 ft.

All the damage rate curves for flying at both altitudes are similar in form to those of Figs. 3 and 4. A slight variation in the characteristic shape is found for some of the components operating up to 12,000 ft. At this altitude some of the damage rate curves are flattened to one side of the peak damage rate, as shown

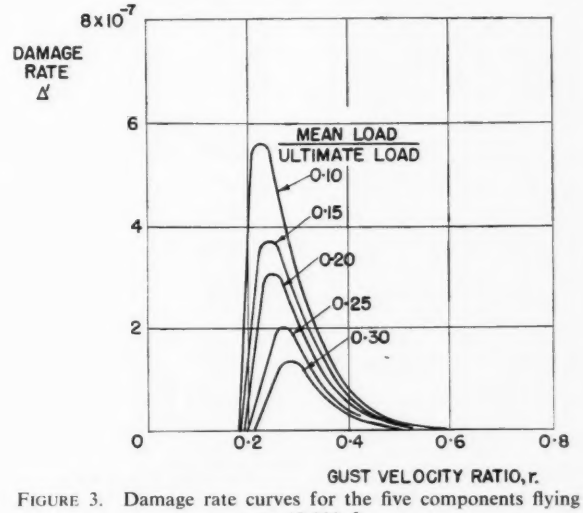

FIGURE 3. Damage rate curves for the five components flying up to 12,000 ft.

TABLE II

DETAILS OF THE THIRTEEN TYPES OF COMPONENT STUDIED

Component	*Brief details of component type*	*Source of reference*
1	Single-row riveted joint, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 4F01, p. 38 (Also Ref. 3)
2	Single-row riveted joint, 17S-T Plate	Unpublished Report ALCOA (Also Ref. 3)
3	Double-row riveted joint, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 4F01, p. 39 (Also Ref. 3)
4	Single-row welded joint, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 3F16, p. 45 (Also Ref. 3)
5	Double-row welded joint, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 4F01, p. 32 (Also Ref. 3)
6	Three-row welded joint, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 4F01, p. 33 (Also Ref. 3)
7	Tension specimens, 24S-T Alclad Sheet	N.A.C.A. A.R.R. 4E30, p. 12 (Also Ref. 3)
8	Riveted lap joint (1), 24S-T Alclad Sheet	Ref. 7
9	Riveted lap joint (2), 24S-T Alclad Sheet	Ref. 7
10	Tension specimens, 24S-T Alclad Sheet	Ref. 7
11	Tension specimens, 24S-T Alclad Sheet	Ref. 7
12	Tension specimens, 75S-T6	Ref. 8
13	Tension specimens, 24S-T4/14S-T6	Ref. 8

in Fig. 5. This flattening effect is due to the influence of a relatively wide range of heavily damaging gust velocities. The effect has no appreciable influence on the general shape of the damage rate curve; no similar flattening of the damage rate curves has been noted for the components flying above 30,000 ft.

The characteristic shape of the damage rate curve suggests that the area under the curve (or the total fatigue damage in each mile of flight) is largely dependent upon the maximum ordinate of the damage rate curve. In fact, for flying up to 12,000 ft. there is a linear relationship given by

$$\Delta = 0{\cdot}25\,\Delta'_{max.}$$

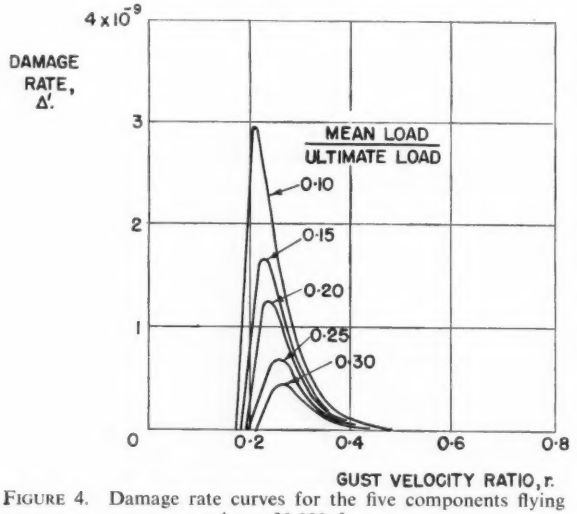

FIGURE 4. Damage rate curves for the five components flying above 30,000 ft.

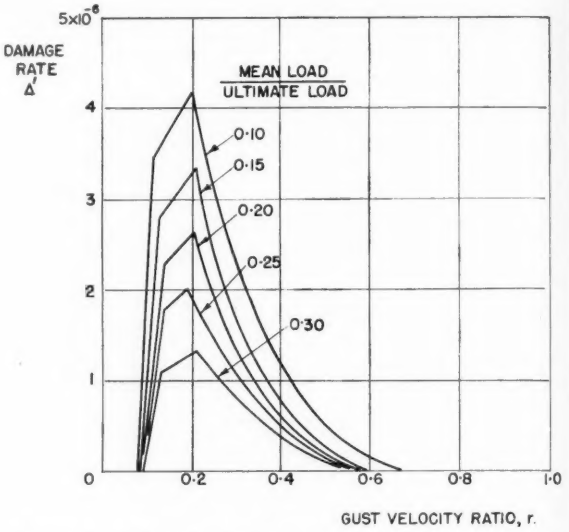

FIGURE 5. "Flattening" effect in some damage rate curves for flying up to 12,000 ft.

as shown in Fig. 6. This implies that the range of gust velocities over which the fatigue damage occurs remains almost constant. The extent of the damage is then governed almost entirely by the maximum ordinate of the damage rate curve.

The gust velocities at which the maximum damage rates occur vary with the total damage in a somewhat irregular fashion, as shown in Fig. 7. There is, however, a general trend suggesting that the most damaging gust velocities are relatively low for structures of low fatigue strength.

Since we have that

$$\Delta' = \frac{d\Delta}{dr} = -\frac{1}{N}\frac{dE}{dr},$$

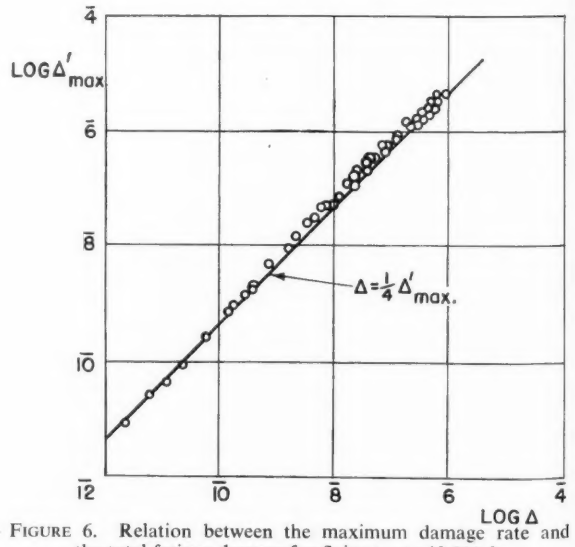

FIGURE 6. Relation between the maximum damage rate and the total fatigue damage for flying up to 12,000 ft.

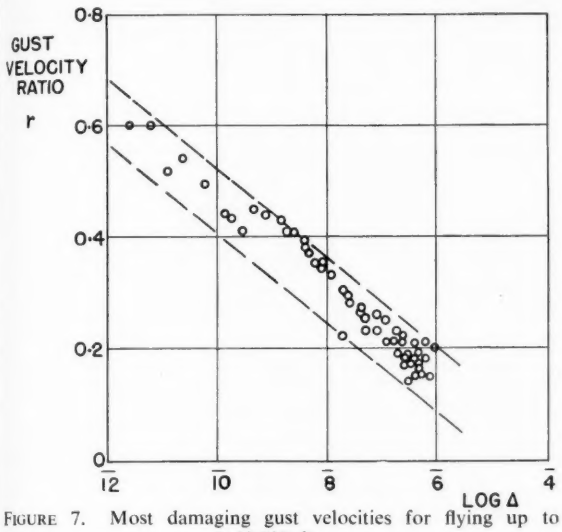

FIGURE 7. Most damaging gust velocities for flying up to 12,000 ft.

Δ' is a function of both $N(r)$ and $E(r)$. It is an interesting feature of all the damage rate curves that the maximum damage rates occur in a limited range of values of $\log N$. For flying up to 12,000 ft. the maximum damage rates for all 65 components lie in the range of values of $\log N$ from 5·5 to 7·5. This means that the form of the r–$\log N$ curve in the same range is important in defining the general shape of the damage rate curve. The average value of $\log N$ for maximum damage date is 6·2; the weaker components tend to have maximum values of Δ' at rather smaller values of $\log N$. For flying above 30,000 ft. all the maximum damage rates occur in the range of values of $\log N$ from 6·0 to 8·0, the average value of $\log N$ being 6·6.

These observations suggest that we may consider the fatigue damage to the component as a function of the gusts giving endurances in the important range of values of $\log N$. Suppose we consider $N=2\times10^6$ cycles as a characteristic value of the endurance; suppose further that r_c is the "characteristic" value of r for which $\log N=6{\cdot}30$ on the r–$\log N$ curve (Fig. 2). Then we find that the total fatigue damage Δ is a well-defined function of r, as shown in Fig. 8. Represented in this form the damage is independent of the ultimate static strength factor, and we need consider only the gust velocity ratio giving an endurance of 2 million cycles.

On comparing the fatigue damage at the two altitudes we observe that for flying above 30,000 ft., the damage per mile of flight is about 1/200th of that for flying up to 12,000 ft. This suggests that for aircraft flying long distances at great height the fatigue damage may be almost negligible, and we may find that the greater part of the fatigue damage is done during the climb and descent.

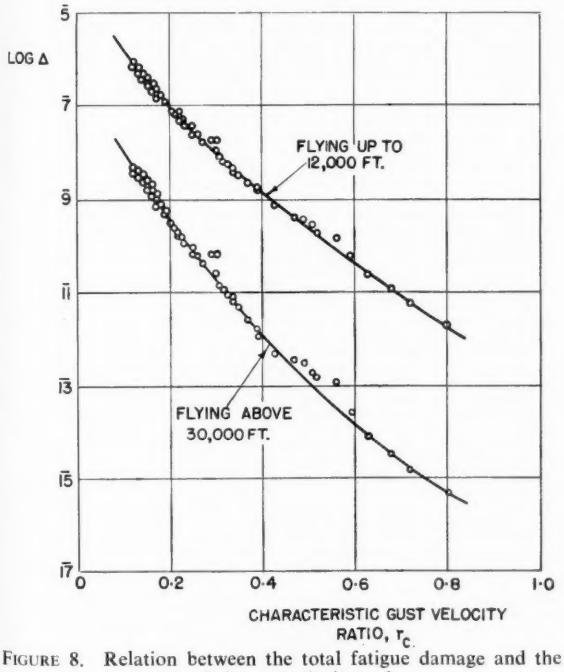

FIGURE 8. Relation between the total fatigue damage and the characteristic gust velocity ratio, r_c.

6. Comparisons with Other Authors

In a recent paper[5] Walker has discussed a number of "criteria" for assessing the fatigue strength of a wing joint under the alternating loads encountered in flight. Each criterion is essentially a test of design skill and is based on extensive experience in the design of wing components. Walker aims at a criterion which will ensure a life for the structure of at least 30,000 flying hours. The requirement for the first criterion is that the component or joint must endure at least 2 million repetitions of an alternating load of $\pm7\frac{1}{2}$ per cent. of the static ultimate superimposed on the mean load of level flight. This criterion takes no account of the actual loads met by the aircraft in flight, and, to improve the criterion, Walker considers the alternating load as modified to $\pm7\frac{1}{2}$ per cent. of the required factored load

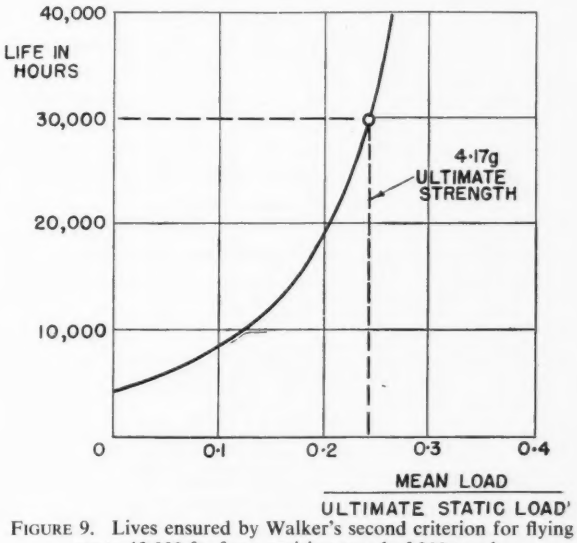

FIGURE 9. Lives ensured by Walker's second criterion for flying up to 12,000 ft., for a cruising speed of 200 m.p.h.

for the 50 ft./sec. gust case. In this latter form the criterion may be compared with the curves of Fig. 8; if r' is the gust velocity ratio corresponding to $7\frac{1}{2}$ per cent. of the required factored load for the 50 ft./sec. gust case, then the second criterion relates only to components for which $r_c \geqslant r'$. The lives in flying hours ensured by the criterion for aircraft flying up to 12,000 ft. and at 200 m.p.h. are shown in Fig. 9. The estimated life of 30,000 hours is completely ensured only for aircraft with ultimate factors below $4{\cdot}17g$. This represents a serious limitation in the range of applicability of the second criterion. A third criterion put forward by Walker conforms more closely to the basic characteristics of the fatigue damage problem. Walker now considers the alternating load to be that corresponding to a 10 ft./sec. gust cycle. The relevance of a criterion of this type is obvious from a consideration of the curves in Fig. 8. If $N \geqslant 2 \times 10^6$ cycles for the alternating load due to a 10 ft./sec. gust cycle, then from Fig. 8 the life ensured up to 12,000 ft. is $1{\cdot}175 \times 10^7$ flying miles, or 58,800 hours at 200 m.p.h. This represents a factor of nearly two on the life of 30,000 hours estimated by Walker.

A further interesting contribution to the problem of fatigue damage has been made by Williams[6] in an article dealing with low strength wing joints. Williams, realising the importance of the r–$\log N$ curve in the region of $N = 2 \times 10^6$, has studied the problem in terms of "fatigue stress factors," which give the alternating loads (as a fraction of the ultimate static strength) which can be endured by the joints for about 2 million cycles. For flying at low altitudes the lives computed by Williams are compared with those estimated by the author in Fig. 10. The comparison is good, considering that Williams has assumed different frequencies of gust loading.

Conclusions

The adoption of a simple cumulative damage law makes possible the evaluation of fatigue damage to a wing joint in a relatively simple manner. A discussion of the problem in terms of infinitesimal intervals of gust loading leads to damage rate curves which indicate immediately those gust velocities causing the greatest damage.

The characteristic form of the damage rate curve makes for considerable simplification of the problem. In fact the fatigue damage sustained by the component during flight is a simple and well-defined function of that single alternating load giving an endurance of 2 million cycles.

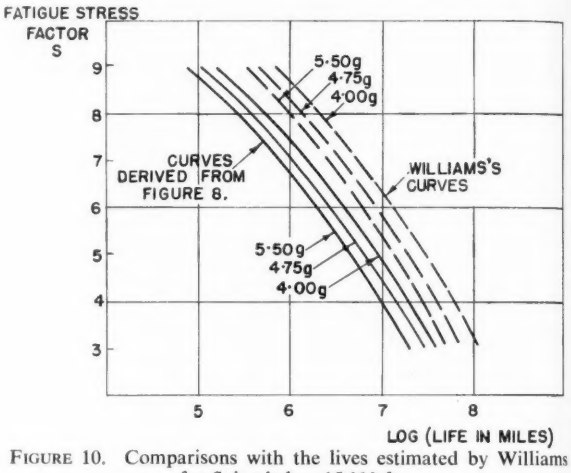

FIGURE 10. Comparisons with the lives estimated by Williams for flying below 15,000 ft.

The fatigue damage over a given flying distance is very dependent upon the altitude of flying. The low velocity gusts at low altitudes cause considerably greater damage than corresponding gusts at high altitude.

The results of the study compare favourably with work put forward by other authors, and the method of analysis adopted may be useful in studying the effects on fatigue damage of a reduction in cruising speed, of changes in the all-up weight of the aircraft, of over-designing the main joints and other factors.

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Recent Developments in the Structural Approach to Aeroelastic Problems

by

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The 899th Lecture to be given before the Royal Aeronautical Society was held on 11th February 1954 at the Institution of Mechanical Engineers, Storey's Gate, London, S.W.1. Mr. G. R. Edwards, C.B.E., F.R.Ae.S., Vice-President of the Society, presided. Introducing the Lecturer, Dr. D. Williams, M.I.Mech.E., F.R.Ae.S., Mr. Edwards said that Dr. Williams was really too well known to need any introduction; his activities had been devoted in recent years to structural problems as applied to aeroplanes. Without reciting his attainments in detail, the job he had tackled in recent years at the Royal Aircraft Establishment and about which he had written papers, covered such subjects as vibration, wing flutter, sandwich construction, the effect of blast on aeroplanes, energy theorems, fatigue, dynamic loading by gusts and landing impacts, water-borne runways and nose-wheel shimmy, a variety of subjects one or all of which had caused most of them a great deal of anguish at one time or another.

In his paper Dr. Williams was going to explain how the electronic digital computing machine would before long make manual computation redundant. He would explain how this machine could be used and also would describe its particular application to aeroelastic problems

1. Introduction

It is a truism to say that no problem in aeroelasticity can be solved without first deriving the elastic characteristics of the structure concerned. In some problems, such as that of determining the critical speed for reversal of aileron control, the elastic properties of the structure under static loads are all that are required in so far as the structure itself is concerned. In most other problems, however, it is necessary to know, in addition, the distribution of mass over the structure—in other words its inertial properties. With these two fundamental properties fully defined, it is then possible to calculate the vibrational characteristics of the structure, *i.e.* its natural frequencies and modes. Having found these, or been presented with them, the aeroelastician needs to know nothing further concerning the structural properties of the aeroplane, his main task thereafter being to derive its response to aerodynamic forces.

In this lecture I propose to discuss only the structural side of the aeroelastic problem, and endeavour to indicate how radically it is being affected by the advent of the electronic digital computing machine—hereafter usually referred to as "the machine."

Although the main theme of this lecture is the profound impact of the electronic digital computing machine on the structural approach to aeroelastic problems, it is my view that the impact on structural analysis —or "stressing" as it is commonly called—is going to be no less profound. For it is obvious that, if the response of a structure to applied loads can be determined accurately enough to form the basis for computing a whole series of frequencies and modes, that same response, giving as it does deflections in terms of loads, constitutes a valuable help towards, if not indeed everything that is required for, stressing the structure. As this is true for every kind of structure used in mechanical and civil engineering, the change which the machine can bring about in man-hours of computation can only be described as revolutionary.

I should here explain that my main object is to expound the general principles of the new approach rather than try to describe or compare the various detailed procedures that are now coming into use. It will be gathered from this that, in preparing this lecture, I have had chiefly in mind the ordinary structural engineer and not the few specialists in this field.

The lecture divides itself naturally into two parts. The first part, by general argument and by a simple example worked out longhand, shows the kind of operations carried out by the machine. And for the reason that the machine likes its data to be provided in the form of matrices, and also that the matrix notation is rapidly becoming the "lingua franca" in the field of aeroelasticity, I have tried to indicate the potentialities of the matrix approach for coping with the kind of problem we are here concerned with.

The second part is intended to show how the principles established in the first part are applied to actual aeroplane structures.

Notation

y_r vertical deflections normal to wing at station r, either total or due only to elastic deformation—according to sense

P_r static force at station r (also used for fin reactions)

U_y strain energy in terms of deflections y
U_P strain energy in terms of static forces P
a_{rs} "stiffness" coefficient = force at station r due to unit displacement at station s with zero displacement maintained at every other station
b_{rs} "influence" or "flexibility" coefficient = deflection at station r due to unit force at station s, with zero force at every other station
ω circular frequency of vibration
A_r amplitude of vibration of displacement y_r at station r
$A_r^{(n)}$ value of A_r in n^{th} vibration mode
m_r effective mass at station r
$g_{rs} = b_{rs} m_s$
l distance of a station from a datum line
l_f distance of fin from reference base
K constraining moment per radian displacement
ϕ angle of roll of reference base
δ deflection of spars and ribs relative to a sloping base, also elastic deflections of fin normal to its plane
α slope of spar or rib relative to a datum line, also used for fin rotation about its base
F_r vertical reaction at station r due to deformation of structure
y'_r displacement at station r directly due to freeing reference triangle
θ angle of pitch of reference triangle about own base
A_0 arbitrary initial value of A_r in iteration process
ψ angle of yaw of reference base
A'_r maximum amplitude of y'_r
z_r displacement of fin at station r normal to plane of fin
z_0 lateral displacement of mid point of base of reference triangle
h_r height of fin station r above fixing base
h_f width of fuselage at base of fin
f_{rs} local influence coefficients for fin deflection relative to encastré fin root
j_{rs} stiffness coefficient for fin stations corresponding to influence coefficients f_{rs}
q_r distance of wing station r from plane of symmetry
B_r maximum total amplitude of z_r
H width of reference base triangle
B'_r amplitude of z_r directly due to freeing base triangle
M mass of wing and fuselage (minus fin)
T_y yawing moment of inertia of wing and fuselage about mid point of base of reference triangle
a_{7r} stiffness coefficients for fuselage station 7 due to fin lateral forces
C_r constants derived from the summation of products

APPENDIX

γ tilting angle of side of box-cell
α, β shear strains in box skin caused by γ
w deflection normal to plane of wing
a shear web pitch
b rib pitch
h depth of wing
t thickness of skin
S shear force in plane of wing skin per unit length
F_r upward reaction at station r necessary to maintain deflections
M_x bending moment per unit length acting on edges parallel to y axis
M_y bending moment per unit length acting on edges parallel to x axis
M_{xy}, M_{yx} twisting moments per unit length of plate edge parallel to the y and x axes respectively
Q_x, Q_y cross-sectional plate shears per unit length of edges parallel to y and x directions respectively
q loading normal to plate per unit area
D plate bending stiffness
ν Poisson's ratio
$S_{r(r+1)}$ average wing (or plate) cross-sectional shear per unit width between stations r and $(r+1)$
$(R_r)_{r(r+1)}$ reaction at station r due to average shear $S_{r(r+1)}$
I moment of inertia of cross-section of thin-wing box-cell per unit width

2. Calculation and Scale-Model

There are two main methods of ascertaining the elastic behaviour of an aeroplane before it is built, one is by calculation and the other is by model experiment. There is nothing to prevent both methods from being adopted for solving the same problem, so making the results obtained by the one method a valuable check on the other. A model good enough to provide reliable values for the lowest half-dozen frequencies and modes cannot be built in much under six months but, once built, it can be used at once for flutter experiments in the tunnel. Changes in mass distribution can also be made very easily and their effect on the flutter characteristics observed directly. Thus, however attractive the approach to aeroelastic problems via machine computation may appear, the approach via a simplified model is not to be lightly set aside. I shall, however, make no further reference in this lecture to the approach via scale models.

3. Degrees of Freedom

Because of the heterogeneous character of the structure of an aeroplane, it is impracticable to write down the general equation governing its vibrational motion. Recourse is therefore had to the universal device of representing the actual structure by one possessing only a limited number of degrees of freedom. The greater the number of the degrees of freedom, the more nearly will the motion of the hypothetical structure conform to that

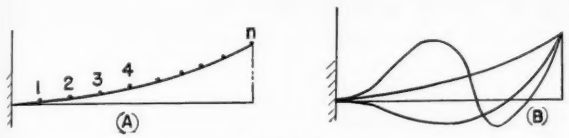


FIGURE 1.

of the actual structure, and it is here that the digital computing machine scores so heavily. Whereas six or eight degrees of freedom were in the past considered an ambitious number to aim at, the use of five to ten times that number is now becoming common with machine help. It has to be remembered that the computational work goes up somewhat more rapidly than the cube of the number of degrees of freedom, to appreciate the advance that this represents.

4. Parameters for Defining Structural Displacement

The parameters in terms of which the displacement of the structure is defined are commonly chosen in one of two ways. These are well illustrated by the simple case of the cantilever beam, the displacement shape of which can be represented either by the independent displacements of a set of points along the beam, as shown in Fig. 1(*a*), or by some combination of the curves (three only shown) of Fig. 1(*b*) defined by the ratios of their amplitudes. It is generally accepted that, for maximum accuracy from a given number of degrees of freedom, the second method holds the advantage. This is natural when it is considered that, with one degree of freedom alone allowed, it is still *possible* for the true displacement shape to be given accurately by a single curve of (*a*) but impossible by the displacement of a single one of the points in (*b*) to the exclusion of displacement at any of the others.

When, however, it is not imperative to restrict the number of degrees of freedom, there is everything to be said for the straightforward simplicity of defining the elastic deformation in terms of the displacements of an adequate number of individual points. The method has the further advantage, which is generally recognised, of being more suitable for application to complex structures like the delta wing, and particularly to structures in which the mass distribution is uneven. Moreover, it is admirably suited to the process of deriving the elastic characteristics of a complete structure like a wing from those of its individual elements—a process that will presently be described.

5. Use of Matrix Methods

The method of successive approximations as carried out by the "machine" requires that the sequence of operations it is asked to perform shall be completely regimented. For this purpose the matrix notation is highly suitable, and therefore a casual acquaintance with that notation is desirable for understanding how the work is partitioned between man and machine. As most structural engineers are unacquainted with matrix algebra, I propose to describe the very few properties of matrices that need to be known for appreciating their use in the new scheme of things.

6. The Two Forms of the Equations of Motion

The two forms in which the equations of motion of a deformable elastic system can be written are well illustrated by the variable section cantilever, whose behaviour under static loading may be considered first.

Figure 2 shows such a cantilever divided into n sections by n stations 1, 2, 3 ..... n, with the actual mass distribution represented as nearly as possible by n discrete masses m_1, m_2, ... m_n, whose displacements are denoted by y_1, y_2, ... y_n.

In the static problem the inertia forces are irrelevant, so forces P_1, P_2, ... P_n acting at the n stations are assumed. The internal energy U can now be expressed in two ways

(*i*) in terms of the deflections y alone, or

(*ii*) in terms of the forces P alone.

The first gives an expression

$$U_y = \tfrac{1}{2}(a_{11}y_1^2 + a_{22}y_2^2 + \ldots + a_{nn}y_n^2) + (a_{12}y_1y_2 + \ldots a_{rs}y_ry_s + \ldots) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the second gives

$$U_P = \tfrac{1}{2}(b_{11}P_1^2 + b_{22}P_2^2 + \ldots + b_{nn}P_n^2) + (b_{12}P_1P_2 + \ldots b_{rs}P_rP_s + \ldots) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These are homogeneous quadratic functions of y and

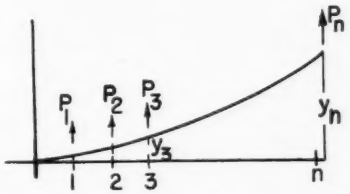


FIGURE 2.

P respectively (in which, since we assume that deflections vary linearly with load, the reciprocal relations $a_{rs} = a_{sr}$, $b_{rs} = b_{sr}$ hold). According to Castigliano's First Theorem

$$\frac{\partial U_y}{\partial y_r} = P_r \quad . \quad . \quad . \quad (3)$$

and according to his Second Theorem

$$\frac{\partial U_p}{\partial P_r} = y_r \quad . \quad . \quad . \quad (4)$$

—two partial differential equations.

Equation (3) is true for non-linear load-deflection relations as well as linear, but equation (4) is true only for linear systems. As, however, all deflections are assumed to be directly proportional to load either (3) or (4) can be used.

By giving r successive values from 1 to n we obtain n equations from (3) which are best visualised in the expanded form

$$\left.\begin{aligned} P_1 &= a_{11}y_1 + a_{12}y_2 + \ldots\ldots + a_{1n}y_n \\ P_2 &= a_{21}y_1 + a_{22}y_2 + \ldots\ldots + a_{2n}y_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ P_n &= a_{n1}y_1 + a_{n2}y_2 + \ldots\ldots + a_{nn}y_n \end{aligned}\right\} \quad . \quad (5)$$

In the same way equation (4) gives

$$\left.\begin{aligned} y_1 &= b_{11}P_1 + b_{12}P_2 + \ldots\ldots + b_{1n}P_n \\ y_2 &= b_{21}P_1 + b_{22}P_2 + \ldots\ldots + b_{nn}P_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n &= b_{n1}P_1 + b_{n2}P_2 + \ldots\ldots + b_{nn}P_n \end{aligned}\right\} \quad . \quad (6)$$

Although one set gives loads in terms of deflections and the other deflections in terms of loads, each set fully defines the elastic characteristics of the cantilever. Indeed each set can be solved to give the other.

7. Equations in Matrix Form

The matrix method of approach may be illustrated right here, both in the matter of notation and use. Thus equation (5) can be written in the form

$$\begin{bmatrix} a_{11}\,a_{12}\ldots\ldots a_{1n} \\ a_{21}\,a_{22}\ldots\ldots a_{2n} \\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots \\ a_{n1}\,a_{n2}\ldots\ldots a_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \ldots \\ \ldots \\ y_n \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \ldots \\ \ldots \\ P_n \end{bmatrix} . \quad (5a)$$

and the way to read this is self-explanatory. The successive terms in the first row of the n-sided *square matrix of stiffness coefficients a* on the left multiply the successive y terms in the adjacent *column matrix of* y's, the sum of the products being equated to the topmost term in the R.H.S. *column matrix of* P's. This gives the first equation in (5); the second and successive equations of (5) are given by repeating the process with the second and successive rows of the square matrix. In matrix jargon, what has been done in (5*a*) is to "*pre-multiply*" a column matrix (the y column) by a square matrix to give another column matrix (the P column).

If there were two columns of variables in the multiplied matrix—the existing y column and a new x column —the y column would be pre-multiplied by the square matrix exactly as before to give the same P column, upon which the same pre-multiplication of the x column would introduce a new P column on the R.H.S. of (5*a*) which would thus become a two-column matrix. It follows at once that an n-columned matrix of variables pre-multiplied by the n-sided square matrix would give n columns on the R.H.S., *i.e.* an n-sided square matrix of P's. In other words the product of two n-sided square matrices is another n-sided square matrix.

So much for matrix notation. The usefulness of the matrix method is equally well illustrated by reference to the two sets of equations (5) and (6). Putting (6) in matrix form, we obtain equation (6*a*)

$$\begin{bmatrix} b_{11}\,b_{12}\ldots\ldots b_{1n} \\ b_{21}\,b_{22}\ldots\ldots b_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots \\ b_{n1}\,b_{n2}\ldots\ldots b_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \ldots \\ \ldots \\ P_n \end{bmatrix} = \begin{bmatrix} \ldots \\ y_1 \\ y_2 \\ \ldots \\ y_n \end{bmatrix} . \quad (6a)$$

where the square matrix of flexibility coefficients b is called a *flexibility matrix* just as the square matrix of stiffness coefficients a in (5*a*) is called a *stiffness matrix.*

As already stated, when equation (5) is solved for the y's the block of "a" coefficients is converted into the block of "b" coefficients of equation (6). The parallel operation in the two matrix equations (5*a*) and (6*a*) is to *invert* the square matrix of a's into the square matrix of b's. The process is called "inversion" by simple analogy with the equation

$$ay = P \quad . \quad . \quad . \quad (7)$$

which has the alternative form

$$y = a^{-1}P. \quad . \quad . \quad . \quad (7a)$$

Thus if $[a]$ represents the "a" matrix and $[b]$ the "b" matrix, $[a]^{-1} = [b]$. Now the important point is that a *simple routine manipulation* of the a matrix will invert it to give the b matrix, and the process can be *done by the machine.*

For the purpose of the lecture this is practically all that need be borne in mind regarding matrix notation and operation. Multiplication has been illustrated in detail but inversion only in what it does—the process itself need not be discussed here.

8. Natural Modes and Frequencies

When the cantilever is vibrating in one of its natural modes at frequency ω, every point on the beam performs a simple harmonic motion so that

$$y_r = A_r \sin \omega t \quad . \quad . \quad . \quad (8)$$

and at full amplitude, if m_r is the associated mass, the inertia force

$$P_r = m_r\omega^2 A_r. \quad . \quad . \quad . \quad (9)$$

By the use of (8) and (9) equations (5) and (6) are at once converted into equations of motion which take the form respectively of (10) and (11).

$$\left.\begin{aligned} m_1A_1\omega^2 &= a_{11}A_1 + a_{12}A_2 + \ldots\ldots + a_{1n}A_n \\ m_2A_2\omega^2 &= a_{21}A_1 + a_{22}A_2 + \ldots\ldots + a_{2n}A_n \\ &\ldots\ \ldots\ \ldots\ \ldots \\ &\ldots\ \ldots\ \ldots\ \ldots \\ m_nA_n\omega^2 &= a_{n1}A_1 + a_{n2}A_2 + \ldots\ldots + a_{nn}A_n \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} A_1 &= b_{11}(m_1\omega^2A_1) + b_{12}(m_2\omega^2A_2) + \ldots\ldots + b_{1n}(m_n\omega^2A_n) \\ A_2 &= b_{22}(m_1\omega^2A_1) + b_{22}(m_2\omega^2A_2) + \ldots\ldots + b_{2n}(m_n\omega^2A_n) \\ &\ldots\ \ldots\ \ldots\ \ldots \\ &\ldots\ \ldots\ \ldots\ \ldots \\ A_n &= b_{n1}(m_1\omega^2A_1) + b_{n2}(m_2\omega^2A_2) + \ldots\ldots + b_{nn}(m_n\omega^2A_n) \end{aligned}\right\}$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

By dividing (11) throughout by ω^2 we obtain

$$\left.\begin{aligned}
\frac{1}{\omega^2}A_1 &= (b_{11}m_1)A_1 + (b_{12}m_2)A_2 + \ldots\ldots + (b_{1n}m_n)A_n \\
\frac{1}{\omega^2}A_2 &= (b_{21}m_1)A_1 + (b_{22}m_2)A_2 + \ldots\ldots + (b_{2n}m_n)A_n \\
&\ldots\quad\ldots\quad\ldots\quad\ldots \\
\frac{1}{\omega^2}A_n &= (b_{n1}m_1)A_1 + (b_{n2}m_2)A_2 + \ldots\ldots + (b_{nn}m_n)A_n
\end{aligned}\right\} \quad (11a)$$

or, in matrix form,

$$\frac{1}{\omega^2}\begin{bmatrix} A_1 \\ A_2 \\ \ldots \\ \ldots \\ A_n \end{bmatrix} = \begin{bmatrix} b_{11}m_1 & b_{12}m_2 & \ldots\ldots & b_{1n}m_n \\ b_{21}m_1 & b_{22}m_2 & \ldots\ldots & b_{2n}m_n \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ b_{n1}m_1 & b_{n2}m_2 & \ldots\ldots & b_{nn}m_n \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \ldots \\ \ldots \\ A_n \end{bmatrix} \quad (11b)$$

in which the square block of coefficients (bm) is called the "*Dynamical Matrix*," incorporating, as it does, both the elastic and the inertial properties of the structure.

9. "Successive Iteration" Process for Obtaining Modes and Frequencies

Where the machine scores is that, apart from the capacity to multiply and invert matrices, it can carry out long iterative processes in astonishingly fast times, and my purpose next is to indicate what it does in this respect (not, however, how it does it).

An equation like (10), treated in ordinary analytical fashion, leads to the usual determinantal frequency equation of power n in ω^2, the $(n-1)$ ratios between the A's being then found for each root ω^2 to give the mode for that frequency. It has long been known, however, that equations of type (10) and (11*a*) can be solved by successive iteration. Taking equation (11*a*) for example, one guesses a particular mode by giving arbitrary values to the n amplitudes of vibration $A_1 \ldots\ldots A_n$ and, for convenience, giving A_n the value of unity, since the ratios of the A's are alone of interest. The equations will not in general be satisfied without giving ω^2 a different value for each.

As it is, the value of $1/\omega^2$ given by the last equation of the set can be taken as a first approximation. On now dividing the quantities obtained on the L.H.S. of the remaining $(n-1)$ equations (obtained from the guessed first set of A's on the R.H.S.) by the value of the first approximation to $1/\omega^2$ just obtained, a new set of A's is derived, in which A_n is still unity. The process is then repeated until finally a value of $1/\omega^2$ and a corresponding set of amplitudes A are found that satisfy all the equations. The whole value of this process of iteration rests on the fact that the successive values of $1/\omega^2$ converge steadily and fairly rapidly towards the correct value (except when two frequencies happen to lie rather close together, when it becomes necessary to make some adjustment to the process in order to hurry the convergence). Once that correct value is reached further iterations obviously merely repeat that value, and when the machine does the iterating the operator can see at once when that stage is reached and sets it going on the next higher harmonic.

It can be shown by a simple proof that this *first value of the root $1/\omega^2$ is at the same time its highest value.* In other words by this method the lowest or fundamental natural frequency is obtained first. If the set of equations (10) had been used (after dividing through by the m's) the highest value of the frequency would have been arrived at first, a fact that makes it *essential to express the elastic characteristics of a structure in terms of a set of deflection (or influence) coefficients—the flexibility matrix*—rather than of a set of stiffness coefficients, or stiffness matrix. Where, for any reason, it is convenient to derive the stiffness matrix first, its inversion must precede the iterative process.

10. Higher Modes and Frequencies

Having found the fundamental mode as a set of n amplitudes $A_1^{(1)}, A_2^{(1)} \ldots\ldots A^{(1)}{}_{n-1}, 1$ (the superscript (1) referring to the first mode), the next higher mode is derived by using the fact that the inertia forces $\omega^2 (m_1A_1, \ldots\ldots m_nA_n)$ associated with a set of amplitudes in any remaining mode do no work in operating over the fundamental set—a direct consequence of the orthogonality of the modes. Thus

$$(m_1A_1)A_1^{(1)} + (m_2A_2)A_2^{(1)} + \ldots + (m_{n-1}A_{n-1})A^{(1)}{}_{n-1} + (m_nA_n)1 = 0 \quad . \quad . \quad . \quad (12)$$

an equation which establishes a relation between the amplitudes A of any vibration from which the fundamental mode has been excluded, or "swept out."

It enables one of the A's (say A_1) to be expressed in terms of the others and so to reduce the number of degrees of freedom by unity. In (11*a*) therefore we can omit the first equation and so, by substituting the value of A_1 given by (12) in the remaining $(n-1)$ equations, be in a position to derive by iteration the highest of the remaining frequencies together with its associated mode, $A_2^{(2)}, A_3^{(2)}, \ldots\ldots A_n^{(2)}$. The value $A_1^{(2)}$ of A_1 in the second mode is given of course by (12) (the superscript (2) denoting the second mode).

In like fashion every remaining mode will be orthogonal both to the first and second modes, and therefore will satisfy not only (12) but also the relation

$$(m_1A_1)A_1^{(2)} + (m_2A_2)A_2^{(2)} + \ldots + (m_nA_n)A_n^{(2)} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

By using (12) and (13) to express A_1 and A_2 in terms of $A_3, A_4, \ldots A_n$ in any remaining mode we can dispense with the *first two equations* of (11*a*) and write the remaining $(n-2)$ equations in terms of $A_3, A_4, \ldots A_n$. Iteration of this set gives the third mode $A_3^{(3)}, A_4^{(3)}, \ldots A_n^{(3)}$ with $A_1^{(3)}$ and $A_2^{(3)}$ then found via (12) and (13).

11. Matrix Method of Iteration

The advantage of the matrix notation is that it enables the iterations just described to be systematised—a great help to the desk computer but a *sine qua non* to the machine. Just as equation (6) gave the matrix

equation (6*a*) so does equation (11*a*) give the matrix equation (14),

$$\begin{bmatrix} g_{11} & g_{12} \cdots\cdots g_{1n} \\ g_{21} & g_{22} \cdots\cdots g_{2n} \\ \cdots\cdots\cdots\cdots\cdots \\ g_{n1} & g_{n2} \cdots\cdots g_{nn} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ .. \\ A_n \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ .. \\ A_n \end{bmatrix} 1/\omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where, for conciseness, coefficients g_{11}, g_{12}, etc., have been written for $b_{11}m_1$, $b_{12}m_2$, etc., in the square Dynamical Matrix.

12. Simple Example

The easiest way to see how the process of matrix iteration works is to consider a very simple example, which can also serve to show how the dynamical matrix itself is formed.

Figure 3 shows a uniform beam $C'C$, whose evenly distributed mass is represented by six equal discrete masses m, pitched a distance l apart at C', B', A', A, B, C. The elastic stiffness of the beam is represented by sprung hinges at the mass stations, each hinge applying a constraining moment $k\theta$ for any angular displacement θ between adjacent segments.

12.1. THE SYMMETRICAL CASE

Consider the symmetrical vibrations of the free-free beam, in which by definition the middle segment $A'A$ remains horizontal but is free to move up and down. By symmetry, only the right-hand side of the beam need be considered.

We first derive the flexibility coefficients b of equation (6), either by first writing down the internal energy U_P and then differentiating with respect to the P's, or directly by the usual and more practical method of writing down the deflections at all stations (here referred to as stations 0, 1 and 2) due to the application of unit load at each station in turn.

With OA (which remains horizontal) as a datum, a unit load applied at station 1 produces deflections b_{11} and b_{12} at stations 1 and 2 respectively of amount l^2/K and $2l^2/K$. Similarly a unit load at station 2 produces deflections b_{21} and b_{22} at stations 1 and 2 of amounts $2l^2/K$ and $5l^2/K$ respectively.

Since the deflections are measured relative to station 0, the static equivalent of equation (6) in the present simple case is

$$\left.\begin{aligned} y_1 - y_0 &= \frac{l^2}{K}(P_1 + 2P_2) \\ y_2 - y_0 &= \frac{l^2}{K}(2P_1 + 5P_2) \end{aligned}\right\} \quad . \quad . \quad (15)$$

In a steady harmonic vibration we can put $y_0 = A_0 \sin \omega t$, $y_1 = A_1 \sin \omega t$, $y_2 = A_2 \sin \omega t$, so that the inertia forces at full amplitude (replacing P_0, P_1, P_2) are respectively $m_0\omega^2 A_0$, $m_1\omega^2 A_1$ and $m_2\omega^2 A_2$.

Substituting these values in (15) gives

$$\left.\begin{aligned} A_1 - A_0 &= \frac{\omega^2 l^2}{K}(m_1 A_1 + 2m_2 A_2) \\ A_2 - A_0 &= \frac{\omega^2 l^2}{K}(2m_1 A_1 + 5m_2 A_2) \end{aligned}\right\} \quad . \quad (16)$$

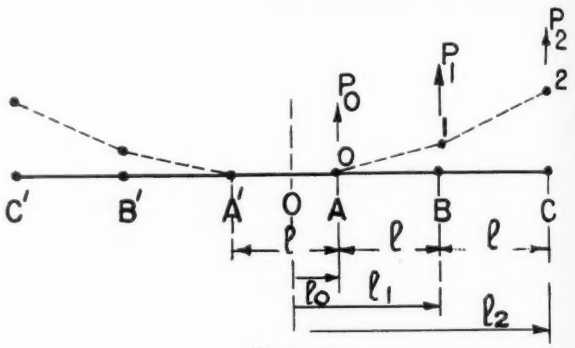


FIGURE 3.

Since there is no resultant vertical force, we can use the condition

$$m_0 A_0 + m_1 A_1 + m_2 A_2 = 0 \quad . \quad . \quad (17)$$

to eliminate A_0 from the L.H.S. of (16), and so give

$$\left.\begin{aligned} \left(1 + \frac{m_1}{m_0}\right) A_1 + \left(\frac{m_2}{m_0}\right) A_2 &= \frac{\omega^2 l^2}{K}(m_1 A_1 + 2m_2 A_2) \\ \left(\frac{m_1}{m_0}\right) A_1 + \left(1 + \frac{m_2}{m_0}\right) A_2 &= \frac{\omega^2 l^2}{K}(2m_1 A_1 + 5m_2 A_2) \end{aligned}\right\} \quad (18)$$

or, since $m_0 = m_1 = m_2 = m$ in the present example

$$\left.\begin{aligned} 2A_1 + A_2 &= \left(\frac{m\omega^2 l^2}{K}\right)(A_1 + 2A_2) \\ A_1 + 2A_2 &= \left(\frac{m\omega^2 l^2}{K}\right)(2A_1 + 5A_2) \end{aligned}\right\} \quad . \quad (19)$$

In matrix notation

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \frac{ml^2\omega^2}{K} \quad . \quad (20)$$

and, as in (7) and (7*a*), we can invert the L.H.S. square matrix to give

$$\begin{aligned} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \frac{ml^2\omega^2}{K} \\ &= \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \frac{ml^2\omega^2}{K} \quad . \quad (21) \end{aligned}$$

which, on carrying out the multiplication of the two matrices on the R.H.S. and bringing the factor $ml^2\omega^2/K$ across gives

$$\left(\frac{3K}{ml^2}\right) \frac{1}{\omega^2} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} . \quad (22)$$

The two equations represented by (22), *i.e.*

$$\left.\begin{aligned} (3K/ml^2\omega^2) A_1 &= 0A_1 - A_2 \\ (3K/ml^2\omega^2) A_2 &= 3A_1 + 8A_2 \end{aligned}\right\} \quad (22a)$$

could have been derived at once from (19) in this simple case, but the process of inversion and multiplication as carried out by the machine saves an enormous amount of labour when there are 30 to 50 variables. This applies even more to the iteration process which is now performed on (22) to derive the two frequencies and modes corresponding to the equation of motion (22*a*).

A value of the quantity $(3K/ml^2\omega^2)$ is required (and hence of ω^2) and the corresponding ratio A_1/A_2 that will satisy (22). As already described, arbitrary values of A_1 and A_2 are taken to start the iteration, say $A_1=1$, $A_2=1$. Substituting these values in the R.H.S. of (22) we have, following the rule for pre-multiplication above explained,

$$\begin{bmatrix}0 & -1\\ 3 & 8\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}-1\\11\end{bmatrix}=11\begin{bmatrix}-1/11\\1\end{bmatrix}.$$

Comparing this with the L.H.S. of (22) it is seen that $(3K/ml^2\omega^2)$, or λ^2 for conciseness, has to have the value 11 if A_2 is to repeat its unit value—this is the first approximation to λ^2. The corresponding mode $A_2:A_1$ is given by the ratio $1:(-1/11)$, a radical change from the initial arbitrary ratio of 1:1. Repeating the process on this new column matrix gives

$$\begin{bmatrix}0 & -1\\ 3 & 8\end{bmatrix}\begin{bmatrix}-1/11\\1\end{bmatrix}=\begin{bmatrix}-1\\85/11\end{bmatrix}=\frac{85}{11}\begin{bmatrix}-11/85\\1\end{bmatrix}=$$
$$=7{\cdot}727\begin{bmatrix}-0{\cdot}1294\\1\end{bmatrix}.$$

Here, starting with a ratio for $A_2:A_1$ of $1:-1/11$ or $1:-0{\cdot}0909$ we end with the ratio $1:0{\cdot}1294$—a 43 per cent. difference. The next iteration gives.

$$\begin{bmatrix}0 & -1\\ 3 & 8\end{bmatrix}\begin{bmatrix}-0{\cdot}1294\\1\end{bmatrix}=\begin{bmatrix}-1\\7{\cdot}612\end{bmatrix}=7{\cdot}612\begin{bmatrix}-0{\cdot}1314\\1\end{bmatrix}$$

where the difference is now only 1·5 per cent. At the next iteration the modal column practically repeats itself,

$$\begin{bmatrix}0 & -1\\ 3 & 8\end{bmatrix}\begin{bmatrix}-0{\cdot}1314\\1\end{bmatrix}=\begin{bmatrix}-1\\7{\cdot}606\end{bmatrix}=7{\cdot}606\begin{bmatrix}-0{\cdot}1315\\1\end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Thus finally the highest value of λ^2 (*i.e.* $3K/ml^2\omega^2$) is 7·606, from which the fundamental frequency

$$\omega^2=3K/ml^2\div 7{\cdot}606=0{\cdot}394K/ml^2. \quad . \quad (24)$$

The corresponding ratio $A_2/A_1=-7{\cdot}606:1$, from which it immediately follows from (17) that $A_0=-(A_1+A_2)=6{\cdot}615$ and therefore the complete mode is given by

$$A_0:A_1:A_2=6{\cdot}615:1:-7{\cdot}606 \quad . \quad (25)$$

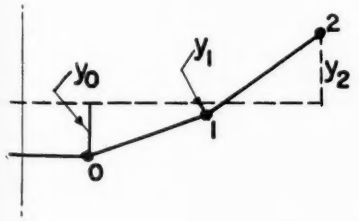
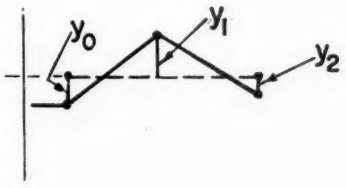


FIGURE 4.

FIGURE 4(*a*).



as shown in Fig. 4. The result, of course, agrees with the mode corresponding to the lower of the two frequencies obtained in the ordinary way from the quadratic in ω^2.

If in (20) we had inverted the R.H. instead of the L.H. matrix we should have been iterating for the highest value of ω^2 and not for $1/\omega^2$ and would then have derived the highest—and in this case the only remaining—frequency. However, the remaining frequency and mode will be derived as if it were the second of a large number, in order to illustrate the normal procedure. The second frequency therefore is obtained as the highest of those remaining after eliminating the fundamental.

Following the argument of Section 10 note that, since the forces m_0A_0, m_1A_1, m_2A_2 in any remaining mode A_0, A_1, A_2 do no work in operating over the displacements $A_0^{(1)}$, $A_1^{(1)}$, $A_2^{(1)}$ of the fundamental, we have the relation (since the m's happen to be equal)

$$A_0A_0^{(1)}+A_1A_1^{(1)}+A_2A_2^{(1)}=0$$

or, using (25),

$$6{\cdot}615A_0+A_1-7{\cdot}606A_2=0. \quad . \quad (26)$$

But, as the resultant force must still vanish, (17) still holds, so that putting $A_0=-(A_1+A_2)$ in (26), we have

$$-5{\cdot}615A_1-14{\cdot}23A_2=0. \quad . \quad . \quad (27)$$

Thus here, by virtue of the second mode being the only remaining mode, the ratio $A_1:A_2$ is found at once as

$$A_2=-0{\cdot}394A$$

and, from (26) or (17), $A_0=-0{\cdot}605A_1$. Thus

$$A_0:A_1:A_2=-0{\cdot}605:1:0{\cdot}394 \quad . \quad (28)$$

a mode shown in Fig. 4(*a*).

The corresponding frequency in this case is at once obtained from either of equations (22*a*) as

$$\omega^2=7{\cdot}615K/ml^2 \quad . \quad . \quad . \quad (29)$$

12.2 THE ANTI-SYMMETRICAL CASE

In the anti-symmetrical case the governing condition is zero bending moment in the mid-section $A'A$ of Fig. 3 and not zero shear as in the symmetrical vibration. The mid-section is now free to rotate about its stationary centre O through any angle ϕ defined by

$$\phi=y_0/l_0 \quad . \quad . \quad . \quad (30)$$

thus producing additional deflections

$$y_0=\phi l_0,\ y_1=\phi l_1,\ y_2=\phi l_2. \quad . \quad . \quad (31)$$

Deformations of the beam are therefore to be measured relative to the straight line defined by the angle ϕ, so that the static equation (15) now becomes

$$\left.\begin{aligned} y_1-\phi l_1&=y_1-\left(\frac{l_1}{l_0}\right)y_0=\frac{l^2}{K}(P_1+2P_2)\\ y_2-\phi l_2&=y_2-\left(\frac{l_2}{l_0}\right)y_0=\frac{l^2}{K}(2P_1+5P_2)\end{aligned}\right\} \quad . \quad (32)$$

while the dynamic equation (16) becomes

$$\left.\begin{aligned} A_1-(l_1/l_0)A_0&=(\omega^2l^2/K)(m_1A_1+2m_2A_2)\\ A_2-(l_2/l_0)A_0&=(\omega^2l^2/K)(2m_1A_1+5m_2A_2)\end{aligned}\right\} \quad (33)$$

Instead of making the resultant vertical force zero, we now make the moment at the centre O zero by writing

$$(m_0A_0)l_0+(m_1A_1)l_1+(m_2A_2)l_2=0 \quad . \quad (34)$$

and hence (with the m's being equal)

$$-A_0=+(l_1/l_0)A_1+(l_2/l_0)A_2. \quad . \quad (34a)$$

Substituting this in (33) after putting $l_1=3l_0$, $l_2=5l_0$, gives

$$\left.\begin{aligned}10A_1+15A_2&=(m\omega^2l^2/K)(A_1+2A_2)\\15A_1+25A_2&=(m\omega^2l^2/K)(2A_1+5A_2)\end{aligned}\right\} \quad (35)$$

or, in matrix form—parallel to equation (20),

$$\begin{bmatrix}10 & 15\\15 & 25\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}=\begin{bmatrix}1 & 2\\2 & 5\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}\frac{ml^2\omega^2}{K}, \quad (36)$$

which can now be iterated in exactly the same way as before.

It is important to note that, by making use of the symmetry of the beam, *i.e.* by fixing the central segment for the purpose of calculating the elastic coefficients, we have been able to split the problem of finding its four frequencies and modes into the two separate and simpler problems of finding, first the two symmetrical, and secondly the two anti-symmetrical, frequencies and modes. If any other segment of the beam had been fixed, there would, after later giving it linear and angular freedom, have been a fourth order instead of a second order matrix to iterate with, and therefore about six to eight times the computational labour.

It is also worth noting that in the absence of symmetry, and so far as the amount of work is concerned, nothing is gained by fixing one segment (or one pair of stations) rather than another since, with a total of n stations, we are always left with $(n-2)$ equations after allowing for linear and angular freedom of the fixed pair of stations (as compared with $(n/2-1)$ when symmetry exists). In practice however, it is naturally better to choose for temporary fixation a part of the structure where the main masses are concentrated.

This concludes the first part of the paper and will provide, I trust, an adequate background for the second part.

13. How to Obtain the Essential Structural Data—the Dynamical Matrix—for Use by the Machine

Faced with a complicated structure like an aeroplane, the aim in the task of deriving the frequencies and modes is to reduce human effort to a minimum while exploiting the machine to a maximum.

To fix ideas, consider the delta wing shown diagrammatically in Fig. 5, where CC' is the fuselage centre line and AA', BB', DD' and FF' represent rigid fuselage frames for carrying four of the spars right across. The other two spars, which are shown dotted, are only pin-jointed to their respective fuselage frames. The spars are crossed by seven ribs R_1, R_2, ... R_7 which run parallel to the plane of symmetry.

To derive the elastic characteristics of this structure we need to choose a reference base that can be imagined to be rigidly held while the deformations of the structure

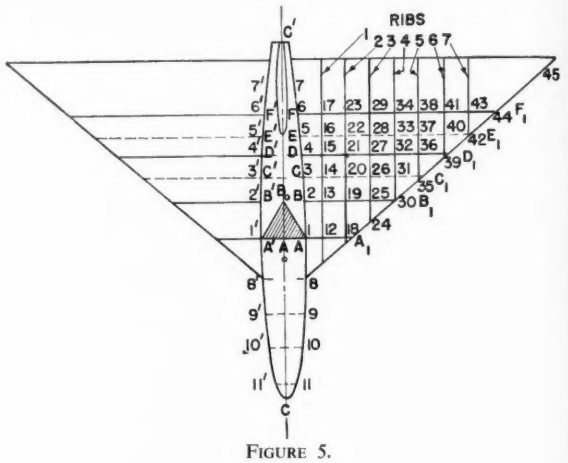


FIGURE 5.

relative to that base are found in the form either of stiffness or flexibility coefficients. Naturally, a base is chosen that is symmetrical about the plane of symmetry so as to exploit the port-starboard symmetry of the structure. Longitudinally it can be fixed anywhere but, as indicated already, a location is favoured near the c.g. where usually the biggest masses lie.

Let the reference base therefore be defined by the three corners of the triangle $AA'B_0$ formed by the intersections of the front spar with the fuselage sides (stations 1, 1′), and the centre B_0 of the straight line BB', joining the corresponding intersections of the second spar. This straight line may be visualised as an imaginary bar connecting B to B', below which the spar itself can freely deflect. Another imaginary bar may be regarded as bridging AA' below which the spar centre can deflect.

In order to confine the function of the reference base to that of merely locating the aeroplane structure in space without causing any interference with its deformation under load, a triangular reference base is chosen, at each corner of which the structure is imagined *simply supported*. Any three points in the structure can be so fixed, subject to their symmetrical disposition relative to the plane of symmetry of the aircraft. Theoretically we could, for example, choose the two wing tips and the tail, but it would not be a good practical choice.

Whether the symmetrical or anti-symmetrical case is considered, account need only be taken of the structure on one side of the plane of symmetry—here the port side. This side is marked with some forty-five stations, most of them located at the intersections of spars and ribs and a few chosen at moderate intervals along the fuselage.

14. Basis of Procedure

As already shown, if we want to start with the fundamental frequency and mode and then work up to the higher values, the matrix that will finally be iterated by the machine must be the dynamical matrix (11*b*), which depends on the flexibility coefficients b. This, at first sight, seems to be a fortunate circumstance, since for

structural elements such as beams it is simpler and more straightforward to calculate the flexibility (*i.e.* influence) coefficients than the stiffness coefficients. For a complicated structure, however, it is a fact (or so I am convinced) that the stiffness coefficients are much more readily calculated than the influence coefficients.

The point is worth emphasising, and this can easily be done by comparing what is involved in calculating the influence coefficients of a variable section cantilever (Fig. 6) on the one hand and the wing of Fig. 5 on the other. In Fig. 6 the deflection and slope at station 1 due to a unit load applied at 1, gives easily calculated clear-cut values for the deflections at all the other stations 2, 3, 4, ... n, and the same applies to the deflections arising from a unit load at any other station. *The reason for this great simplicity is that simple equilibrium conditions are adequate to determine all the deflections wherever the unit load is applied.*

Now compare this with a unit load applied, say, at station 28 of the wing of Fig. 5. It is seen at once that it is resisted by a maze of interconnecting members and that to calculate their respective contributions is to undertake the resolution of a tangle of redundancies. For certain straightforward wing-fuselage layouts this method of approach has not been without its advocates in the past, but it is now fairly well recognised that, applied to highly redundant complicated structures, it is not to be recommended.

Thus, although it is the dynamical matrix built up of influence coefficients that is finally needed for the iteration process, it is to be computed not directly but by inverting the stiffness matrix. In other words we aim at expressing the elastic characteristics of the wing-fuselage structure by specifying the force at each station in terms of hypothetical displacements (relative to a temporarily fixed base) at all the stations.

15. The Symmetrical Case for the Delta Wing

To demonstrate the procedure, consider the symmetrical case for the delta wing of Fig. 5. For this case assume the structure to be firmly held by three simple supports at the points A_1, B_0, A', the deflections of which will therefore be zero. Since the straight line BB' has no angular displacement in roll for symmetrical vibrations, fixing B_0 in this case fixes also B and B'. All the other numbered stations will each have its own deflection, the spar BB_1, for example, having deflections 0, y_{13}, y_{19}, y_{25} at the four stations 2, 13, 19 and 25. Indeed the whole wing may be regarded as being forced to take up an arbitrary contour defined by the system of deflections y (positive upward to fix ideas), so that the deformation of the fuselage and of every spar, rib and

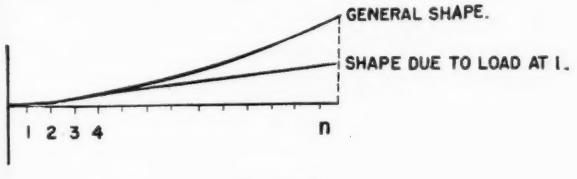


FIGURE 6.

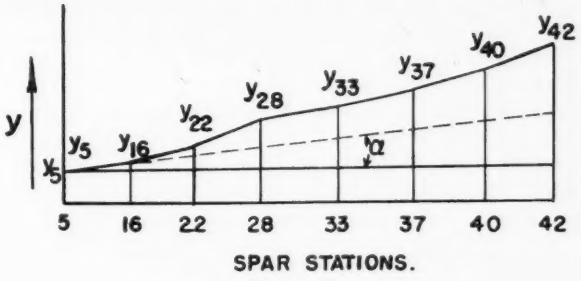


FIGURE 7.

torsion box is defined in terms of the y's. Each of these structural members will register its resistance to this deformation by setting up reaction forces at the particular stations that govern its deformation. Moreover, it is clear that, apart from members that are directly connected to the fixed base, every structural member must be in equilibrium under the constraining reaction forces.

This suggests that if, at each station, we find the reaction set up by every individual member—rib, spar and torsion box—that is affected by the deflection at that station, and then add them all algebraically, we shall have derived the resultant reaction *at that station* due to the original arbitrary contour of y's. This, in point of fact, is the essence of the procedure, which I shall now discuss in more detail.

16. Individual Members of Beam Type

Certain individual members—spars and ribs and even the fuselage itself—can be regarded as beams forced to take up specified deflections. As a typical example, consider the spar EE_1, which is pin-jointed to the fuselage at E, and is forced into the deflection curve defined by the 8 deflections y_5, y_{16}, y_{22}, y_{28}, y_{33}, y_{37}, y_{40} and y_{42}, as in Fig. 7. The resistance set up by the spar is defined by the deflections at the outer six stations relative to the base formed by the inner two, *i.e.* the deflections relative to the dotted base of Fig. 7. The spar may therefore be regarded as simply supported at stations 5 and 16 and subject to deflections $\delta_1, \delta_2 \ldots \delta_6$ at the other six stations. If the distances of the successive stations from the root station 5 are $l_1, l_2 \ldots l_7$, and if the angle $(y_{16} - y_5)/l_1$ is called α, the values of these δ's are given by

$$\delta_1 = y_{22} - (y_5 + l_2\alpha), \quad \delta_2 = y_{28} - (y_5 + l_3\alpha) \ldots$$
$$\ldots \delta_6 = y_{42} - (y_5 + l_6\alpha). \quad . \quad . \quad (37)$$

The problem now is to determine the six reactions necessary to hold the arbitrary deflections δ relative to the temporarily fixed base defined by the first two stations. We want, in other words, forces in terms of deflections, *i.e.* the appropriate stiffness coefficients. For a beam however, as already indicated, it is much easier to derive the influence than the stiffness coefficients, and this is where the machine again helps. For not only can it be used to perform the tedious task of computing the flexibility matrix (from simple basic data such as the mass and the bending flexibilities of the various segments of the beam) but it can, in turn, invert that

matrix to give what is really required—the stiffness matrix. From this we read off the 6 reaction forces induced at the 6 outer stations in terms of the 6 δ's and hence, by (37), in terms of the y's.

It remains to determine the reactions at the temporary base stations 5 and 16 by using the fact that the whole set of 8 reactions must be self-equilibrating. Their algebraic sum must therefore be zero and so must be the sum of their moments about any convenient point (one of the two base stations for example). Thus the 8 forces P_5, P_{16}, P_{22}, P_{28}, P_{33}, P_{37}, P_{40} and P_{42} are finally obtained in terms of the 8 deflections $y_5, \ldots y_{42}$.

The same method exactly is applied to the leading edge member (since the leading edge forms a shear web) and to each of the ribs in turn, but a spar that crosses or bridges the fuselage needs no such temporary fixing base as that just described for a spar pinned at the root, because its equilibrium is ensured by its fixed root. Should, however, the spar pass clean through the fuselage, its centre will deflect relative to the two fuselage intersections and the most convenient way of dealing with such a case is to introduce an extra station at its mid point, where the spar can then be regarded as encastré.

The fuselage itself can also be regarded as a simple beam held at the three fixed points A, B_0, A' and constrained to follow the arbitrary deflection curve specified by the 10 deflections—y_2, y_3, y_4, y_5, y_6, y_7 aft of the front spar and y_8, y_9, y_{10} and y_{11} forward of that base.

17. Individual Torsion Boxes

The four intersections of a pair of adjacent spars and ribs such as the four points 21, 22, 28, 27 in Fig. 5 mark the four corners of a torsion box. The box is regarded as undeformed, and therefore requiring no constraining forces at its corners, if those corners lie in a plane. Deformation out of that plane, however, brings into action four self-equilibrating corner reactions which can be expressed in terms of the four corner deflections (*see* Appendix Part I).

I have here dealt with no more than the basic idea of the principle of splitting up the structure into well-defined individual members. This is because any detailed discussion of the many questions that arise at this stage is better considered later when it cannot interfere with the general argument. A major question is whether there may not be a better and simpler method of finding the elastic coefficients than by such splitting-up, and in Part II of the Appendix I have outlined a possible new approach that may answer this question.

Suppose now that the resultant reaction has been determined at every station in terms of the original set of y's. At station 21, for example, there is a resultant reaction F_{21} made up of a spar and a rib reaction and the four reactions from the four torsion boxes that meet at station 21. It is expressed in terms of y_{21} itself, 6 spar y's, 5 rib y's and the 8 y's of the boxes that have station 21 in common—20 deflections in all. Thus, when the equation for F_{21} is written in terms of the deflections, each with its own stiffness coefficient, only 20 of the 45 y's will have non-zero coefficients.

The complete set of equations will now have this appearance:—

$$\left.\begin{aligned} F_3 &= a_{3,3}y_3 + a_{3,4}y_4 + \ldots\ldots\ldots\ldots + a_{3,n}y_n \\ F_4 &= a_{4,3}y_3 + a_{4,4}y_4 + \ldots\ldots\ldots\ldots + a_{4,n}y_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ F_n &= a_{n,3}y_3 + a_{n,4}y_4 + \ldots\ldots\ldots\ldots + a_{n,n}y_n \end{aligned}\right\} \quad (39)$$

or, in matrix form

$$\begin{bmatrix} F_3 \\ F_4 \\ \cdots \\ \cdots \\ F_n \end{bmatrix} = \begin{bmatrix} a_{33}a_{34} \ldots a_{3n} \\ a_{43}a_{44} \ldots a_{4n} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ a_{n3}a_{n4} \ldots a_{nn} \end{bmatrix} \begin{bmatrix} y_3 \\ y_4 \\ \cdots \\ \cdots \\ y_n \end{bmatrix} . \quad . \quad (39a)$$

As however it is deflections in terms of forces and not forces in terms of deflections that are required for the iteration process, the machine is now used to invert the square matrix of stiffness coefficients a_{rs} into the square matrix of flexibility coefficients b_{rs} to obtain the equation

$$\begin{bmatrix} y_3 \\ y_4 \\ \cdots \\ \cdots \\ y_n \end{bmatrix} = \begin{bmatrix} b_{33}b_{34} \ldots b_{3n} \\ b_{43}b_{44} \ldots b_{4n} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ b_{n3}b_{n4} \ldots b_{nn} \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \\ \cdots \\ \cdots \\ F_n \end{bmatrix} . \quad . \quad (40)$$

Note that, *with base ABA'B' fixed*, this gives the deflections of the structure for any *arbitrary* set of loads F, *a result that, incidentally, goes far towards enabling the whole structure to be stressed for static strength purposes.*

The next step is to free the temporarily fixed base and to express the elastic displacements of equation (40) relative to the plane defined by the released base. If, therefore, y'_3, $y'_4 \ldots y'_n$ represent the deflections due to displacements of the base, and if we wish to represent the total displacements by $y_3, y_4 \ldots_n$, we must in (40) put $(y_3 - y'_3)$ for y_3, $(y_4 - y'_4)$ for y_4, etc. On this understanding we write (40) in the form

$$\begin{bmatrix} (y_3 - y'_3) \\ (y_4 - y'_4) \\ \cdots\cdots \\ \cdots\cdots \\ (y_n - y'_n) \end{bmatrix} = \begin{bmatrix} b_{33} \ldots b_{3n} \\ b_{43} \ldots b_{4n} \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ b_{n3} \ldots b_{nn} \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \\ \cdots \\ \cdots \\ F_n \end{bmatrix} . \quad (41)$$

—still the same static equation with arbitrary values of F.

18. Mass Distribution

Before converting this set of static equations into equations of motion each station must be associated with a discrete mass, so that the system of masses thus obtained will represent as nearly as possible the actual mass distribution. In this way the quantity m_r can be taken to represent the mass associated with station r.

Putting now $A_r \sin \omega t$ for y_r and the inertia force $m_r A_r \omega^2 \sin \omega t$ for F_r, we can write (41) in the form

$$\left(\frac{1}{\omega^2}\right)\begin{bmatrix}(A_3 - A'_3)\\(A_4 - A'_4)\\\cdots\\\cdots\\(A_n - A'_n)\end{bmatrix} = \begin{bmatrix}b_{33}m_3, b_{34}m_4 \ldots\ldots b_{3n}m_n\\b_{43}m_3, b_{44}m_4 \ldots\ldots b_{4n}m_n\\\cdots\\\cdots\\b_{n3}m_3, \ldots\ldots b_{nn}m_n\end{bmatrix}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where the square matrix on the R.H.S. is the Dynamical Matrix required for the iteration process.

Amplitudes ($A'_3 A'_4$, etc.) due to displacements of base.

When the structure is vibrating in space there is of course no resultant force at the c.g., in any direction, nor is there a resultant moment about any axis through the c.g., and what is true of the c.g. must be true for every other point. The inertia forces must therefore satisfy the six equations of equilibrium of forces along, and moments about, any three orthogonal axes parallel (for convenience) with the principal axes (say) Ox, Oy and Oz of Fig. 8, where Ox is fore-and-aft and Oy is vertical.

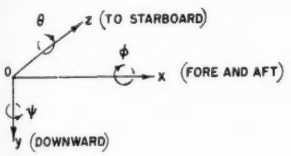


FIGURE 8.

Thus, any inertia forces or moments not automatically in balance with the base fixed must be balanced by the inertia forces introduced as a result of releasing the base.

In the present symmetrical case all the inertia forces are vertical so we need not trouble about the balance of forces along x and z. Moreover, by symmetry, the vertical forces are in balance about the x axis and cannot have any moments about the y axis. We have, however, to balance the forces in the y direction and the moments about an axis parallel to Oz, so we give the base two degrees of freedom—vertical translation and a pitching angle.

In Fig. 5 therefore we imagine the two stations 1, 1′ (at A and A') to have equal vertical amplitudes of oscillation A_1 and that the straight line BB' while having the amplitude A_1 in common, swings about AA' through an angular amplitude θ. This gives every station r an extra amplitude due to base mobility of amount

$$A'_r = A_1 + l_r\theta \quad . \quad . \quad . \quad (43)$$

where l_r is here the *fore-and-aft distance* between station r and station 1 (positive when r is aft of 1). It follows from the definition that

$$\theta = (A_2 - A_1)/l_2 \quad . \quad . \quad . \quad (44)$$

and therefore

$$A'_r = A_1 + (A_2 - A_1) l_r/l_2. \quad . \quad . \quad (43a)$$

The values of A_1 and A_2 of the base points are now to be found from the two equilibrium conditions

(*i*) The resultant of all the vertical forces (including now the base stations) must be zero,

(*ii*) The resultant of their pitching moments must be zero about *any axis* parallel to the z axis and hence about the line AA'.

The two equations that give us A_1 and A_2 are therefore (after dropping the common factor ω^2)

$$m_1A_1 + m_2A_2 + m_3A_3 + \ldots\ldots + m_nA_n = 0 \quad (45)$$

and

$$m_2A_2l_2 + m_3A_3l_3 \ldots\ldots\ldots\ldots + m_nA_nl_n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

(the l's being measured positive for stations aft of the line AA').

These equations give us A_1 and A_2 and hence, by (43*a*), every amplitude A'_r in terms of the remaining A's, *i.e.* $A_3, A_4, \ldots A_n$.

On now substituting the values thus found for $A'_3 A'_4 \ldots A'_n$ in (42), we convert the L.H.S. matrix into a full square matrix of $A_3 A_4 \ldots A_n$, so that we can write (42) in the form

$$\frac{1}{\omega^2}\begin{bmatrix}c_{33}c_{34}\ldots c_{3n}\\c_{43}c_{44}\ldots c_{4n}\\\cdots\\\cdots\\c_{n3}c_{n4}\ldots c_{nn}\end{bmatrix}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} = \begin{bmatrix}b_{33}m_3b_{34}m_4\ldots b_{3n}m_n\\b_{43}m_3b_{44}m_4\ldots b_{4n}m_n\\\cdots\\\cdots\\b_{n3}m_3b_{n4}m_4\ldots b_{nn}m_n\end{bmatrix}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} \quad . \quad (47)$$

The next step—to be carried out by the machine—is to invert the L.H.S. matrix preparatory to moving it across as a multiplier on the R.H.S., thus giving

$$\frac{1}{\omega^2}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} = \begin{bmatrix}d_{33}d_{34}\ldots d_{3n}\\d_{43}d_{44}\ldots d_{4n}\\\cdots\\\cdots\\d_{n3}d_{n4}\ldots d_{nn}\end{bmatrix}\begin{bmatrix}b_{33}m_3\ldots b_{3n}m_n\\b_{43}m_3\ldots b_{4n}m_n\\\cdots\\\cdots\\b_{n3}m_3\ldots b_{nn}m_n\end{bmatrix}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47a)$$

The machine multiplies the two square matrices to give a square product-matrix of the same order thus finally producing, ready for the iteration process, the matrix equation.

$$\frac{1}{\omega^2}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} = \begin{bmatrix}e_{33}e_{34}\ldots\ldots e_{3n}\\e_{43}e_{44}\ldots\ldots e_{4n}\\\cdots\\\cdots\\e_{n3}e_{n4}\ldots\ldots e_{nn}\end{bmatrix}\begin{bmatrix}A_3\\A_4\\\cdots\\\cdots\\A_n\end{bmatrix} \quad . \quad (48)$$

The successive frequencies and modes are now found by the machine by pre-multiplying an arbitrary initial set of values $A_3^{(0)}$, $A_4^{(0)} \ldots A_n^{(0)}$ by the square matrix until the column repeats itself as already described in Part I. As each set of values of A_3 to A_n for a particular mode are found the corresponding pair of amplitudes A_1 and A_2 are derived from (45) and (46).

19. The Anti-symmetrical Case

There is no need to consider this case in detail, but a few remarks may be worthwhile. The most important point to realise is that the great simplification consequent upon using the symmetry of the structure to reduce the number of stations by half, has to some extent to be paid for. The payment takes the form of modifying certain of the fuselage stiffness coefficients and the introduction of some fresh coefficients, in equation (39). The reason for this is that, in the anti-symmetrical case, the reaction forces necessary to hold the series of deflections at stations 8, 9, 10, 11 and 2, 3, 4, 5, 6, 7 depend on the torsional stiffness of the fuselage and not on its bending stiffness. Station 2 is included because at the second spar there is now only to be one fixed station, *i.e.* the (vertically) undeflected central station B_0 at which the spar is effectively pin-jointed, thus allowing the stations 2, 2′ (at B, B') to take up equal and opposite deflections. Before discussing this further it is well to consider what degrees of freedom have to be assigned to the base triangle AB_0A' to satisfy the overall equilibrium conditions for the inertia forces in the anti-symmetrical case. The operating forces and their effects are as follows (with axes as in Fig. 8):

(*i*) *Along the x axis*—no applied forces.
(*ii*) *Along the y axis*—the sets of y forces on either side of the x axis balance and therefore no resultant.
(*iii*) *Along the z axis*—unbalanced z forces (on the fin due to roll) have to be in equilibrium.
(*iv*) *About the x axis*—y forces produce rolling moments that have to be in equilibrium.
(*v*) *About the y axis*—z forces produce yawing moments that have to be in equilibrium.
(*vi*) *About the z axis*—y forces balance out.

It is seen that the inertia forces automatically satisfy equilibrium conditions (*i*), (*ii*) and (*vi*) but must be specially related to satisfy (*iii*), (*iv*) and (*v*), *i.e.* to have zero resultant force *along* the z axis and zero resultant moments *about* the x and y axes. The base triangle must therefore be given linear freedom along the z axis and angular freedom to roll about the x axis and yaw about the y axis. The point A_0 in the triangle can therefore be displaced laterally parallel to the z axis by an amount z_0, yaw through an angle ψ about the vertical axis through A_0 and roll through an angle ϕ about the fore and aft axis through A_0B_0.

It can now be seen that, in obtaining the stiffness coefficients (with the base triangle fixed) it is necessary to introduce z deflections at each fuselage station and also to introduce a number of stations on the fin to be given z deflections.

20. Introduction of Fin Stations

In the present example—the delta wing—the fuselage is prevented from bending laterally by the great rigidity of the wing in its own plane, so that the only elastic displacements in the z direction are those of the fin stations. The influence coefficients for these are probably well enough obtained directly by calculating the displacements of the fin as a simple beam mounted on the base formed by the anti-symmetrically placed fuselage stations 7 and 7′, distant h_t apart as shown in Fig. 9(*a*) and (*b*).

The procedure for taking account of the fin displacements is much the same as that already used for other individual members, except that now new stations have to be added to an existing set. Suppose five stations are taken along the fin span—1, 2, ... 5, and their total lateral displacements (relative to the fixed triangular base) are denoted by $z_1, z_2 \ldots z_5$. Each displacement z_r consists, as shown in Fig. 9(*b*), of a rigid-body displacement αh_r (of the fin relative to the fuselage) and an elastic displacement

$$\delta_r = (z_r - \alpha h_r) \quad . \quad . \quad . \quad (49)$$

where α is the angle through which the base, defined by stations 7, 7′, is displaced. Since the mid-point T has zero vertical deflection (for anti-symmetrical vibrations)

$$\alpha = 2y_7/h_t. \quad . \quad . \quad . \quad (50)$$

The elastic z deflections in terms of arbitrary applied z forces $P_1P_2 \ldots P_5$ (to distinguish them from vertical forces F) may now be written as

$$\left.\begin{aligned} \delta_1 &= f_{11}P_1 + f_{12}P_2 \ldots f_{15}P_5 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \delta_5 &= f_{51}P_1 + \ldots\ldots\ldots f_{55}P_5 \end{aligned}\right\} \quad . \quad . \quad (51)$$

Putting this into matrix form we obtain, on inverting the matrix, the P's in terms of the δ's. Thus

$$\left.\begin{aligned} P_1 &= j_{11}\delta_1 + j_{12}\delta_2 \ldots\ldots + j_{15}\delta_5 \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ P_5 &= j_{51}\delta_1 + \ldots\ldots\ldots + j_{55}\delta_5 \end{aligned}\right\} \quad . \quad . \quad (52)$$

where the stiffness coefficients j have replaced the flexibility coefficients f.

Substituting the values of δ given by (49) and (50), we have

$$\left.\begin{aligned} P_1 &= j_{11}z_1 + j_{12}z_2 + \ldots\ldots + j_{15}z_5 + j_{17}y_7 \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ P_5 &= j_{51}z_1 + j_{52}z_2 + \ldots\ldots + j_{55}z_5 + j_{57}y_7 \end{aligned}\right\} \quad . \quad (53)$$

The reaction at station 7 required to equilibrate the P's is then given by

$$(F_7)_{\text{fin}} = -\frac{1}{h_t}(h_1P_1 + h_2P_2 + \ldots + h_5P_5) \quad . \quad (54)$$

(the negative sign denoting a downward reaction) or, after substituting for the P's from (53)

$$(F_7)_{\text{fin}} = a_{71}z_1 + a_{72}z_2 + \ldots + a_{75}z_5 + a_{77}y_7; \quad . \quad (55)$$

where a dot indicates a station on the fin.

Thus the new forces that have now to be introduced into the matrix of (39) (already modified, let us suppose, to take account of zero bending of the individual spars where they cross the plane of symmetry and to allow for torsion instead of bending of the fuselage) are $P_1, P_2 \ldots P_5$ and the extra force $(F_7)_{\text{fin}}$ at fuselage station

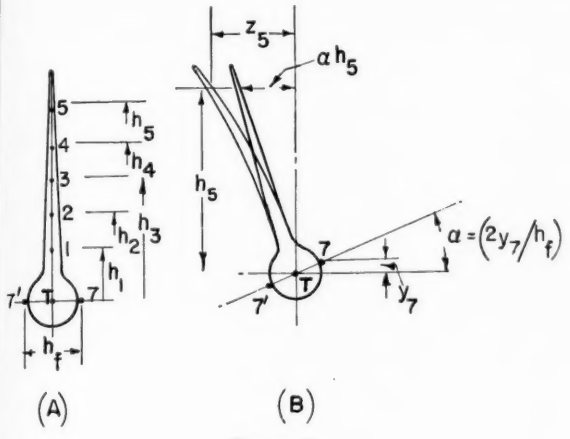


FIGURE 9.

7. These six forces are expressed in equations (53) and (55) in terms of the six displacements z_1, z_2 ... z_5 and y_7. Their introduction into the matrix of (39), means five extra rows and five extra columns in the square matrix but nearly all the new elements will be ciphers, since the P's are independent of all y's except y_7. To make this clear, the modified form of (39) is best shown fully as equation (56)

$$\begin{bmatrix} F_3 \\ F_4 \\ \dots \\ F_7 \\ \dots \\ \dots \\ F_n \\ \hline P_1 \\ P_2 \\ \dots \\ P_n \end{bmatrix} = \left[\begin{array}{cccccc|ccccc} a_{33}a_{34} \dots & & & & a_{3n} & 0 & 0 & 0 & 0 & 0 \\ a_{43}a_{44} \dots & & & & a_{4n} & 0 & 0 & 0 & 0 & 0 \\ \dots & & & & & \dots & & & & \\ a_{73}a_{74} \dots & (a_{77}+a_{77}) & \dots & & a_{7n} & a_{71}a_{72} & \dots & & & a_{75} \\ \dots & & & & & \dots & & & & \\ \dots & & & & & \dots & & & & \\ a_{n3}a_{n4} \dots & & & & a_{nn} & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 \dots & j_{17} & \dots & 0 & j_{11} & j_{12} & \dots & & j_{15} \\ 0 & 0 \dots & j_{27} & \dots & 0 & j_{21} & j_{22} & \dots & & j_{25} \\ \dots & & & & & \dots & & & & \\ 0 & 0 \dots & j_{57} & \dots & 0 & j_{51} & j_{52} & \dots & & j_{55} \end{array}\right] \begin{bmatrix} y_3 \\ y_4 \\ \dots \\ y_7 \\ \dots \\ \dots \\ y_n \\ \hline z_1 \\ z_2 \\ \dots \\ z_5 \end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Two chain dotted lines partition the matrix into four parts of which the left-hand top part constitutes the original square matrix of (39) unchanged except for the one coefficient a_{77} which is now increased by the amount a_{77} (from equation (55)). The R.H.S. top part is filled with noughts except for the F_7 row, which gives the forces at station 7 due to the five fin displacements (equation (55)). The bottom R.H.S. part is the block of fin coefficients of equation (53) except for the last column of (53) which takes its place as the only non-zero column in the L.H.S. bottom part.

This matrix (of which the square block forming the top L.H.S. constitutes by far the greater part) is inverted to give the counterpart of equation (40) for static equilibrium. This gives rise to the counterpart of equation (41), which in turn, if we make $y_r = A_r \sin \omega t$, $z_r = B_r \sin \omega t$, and associate suitable masses with the new stations, gives us the counterpart of equation (42). Here the wing-displacement terms A' on the L.H.S., and the corresponding B' terms for the fin, must take account of the vibration amplitude due to the displacements z_0, ψ and ϕ of the base triangle, just as in equation (42) account was taken of vertical translation and pitch.

We thus have, for the wing stations

$$A'_r = q_r \phi \quad . \quad . \quad . \quad (57)$$

where $\phi = 2A_1 / H$
q_r = distance of wing station r from the plane of symmetry
and H = distance AA_1 between stations 1 and 1' (58)

Also, for the fin stations,

$$B'_r = z_0 + h_r \phi + l_r \psi \quad . \quad . \quad (59)$$

where ψ is the yawing angle of the whole delta wing about the vertical axis through A_0.

There are now three equations of equilibrium for eliminating z_0, ϕ and ψ, just as there were equations (45) and (46) to eliminate y_0 and θ in the symmetrical case. The three are:—

For zero moment in roll—

$$(m_1 q_1 A_1 + m_2 q_2 A_2 + \dots m_n q_n A_n) + + (m_{1f} h_1 B_1 + m_{2f} h_2 B_2 + \dots + m_{5f} h_5 B_5) = 0, \quad . \quad (60)$$

where suffix f stands for fin.

For zero moments in yaw—

$$I_y \psi + (m_{1f} B_1 + m_{2f} B_2 + \dots + m_{5f} B_5) l_f = 0, \quad . \quad (61)$$

where I_y = yawing moment of inertia of wing and fuselage about station A_0, and l_f = distance from A_0 to c.g. of fin.

For zero linear force along z—

$$M z_0 + (m_{1f} B_1 + m_{2f} B_2 + \dots m_{5f} B_5) = 0, \quad . \quad (62)$$

where M = mass of wing and fuselage (without fin).

From (60) we obtain A_1 (in terms of all the other A's and the B's) and hence ϕ by (58) and A'_r by (57). From (61) we obtain ψ (in terms of the B's), and from (62) z_0 (in terms of the B's), and hence by (59) and B'_r's.

Thus, having got in the anti-symmetrical case to the stage of equation (42) for the symmetrical case, the values of A'_r and B'_r given by (57) and (59) in the L.H.S. matrix are substituted to give a square matrix like the L.H.S. of (47). One matrix inversion and one multiplication gives finally an equation that is the counterpart of (48), in which form it is ready for the iteration process that is to give the series of natural frequencies and modes that are required.

20. A word about the Matrix Notation

It is, I think, worth emphasising again that, for the purpose of making use of the machine to obtain the natural frequencies and modes of a structure, next to nothing need be known about the theory of matrices. What little is required however is essential and for this two sources may be consulted—the brief section by von

Kármán and Biot on matrix iteration in their book[1], and Frazer, Duncan and Collar's well known book[2].

21. Practical Experience of Method Described

At present the method of deriving the dynamical characteristics of a structure by representing its deformation under load by the deflections at a comparatively large number of point stations is in its infancy. This is because the only apparatus that makes such an approach feasible—the high-speed digital computer—has only very recently made its appearance as a reliable piece of machinery. It is not surprising therefore that opportunities for comparing results obtained with the machine with test results for the actual structure are somewhat rare.

It is obvious, of course, that the accuracy with which the machine turns out the frequencies and modes is entirely dependent on the accuracy with which the influence (or stiffness) coefficients are worked out in the first place. Any lack of agreement therefore between computed and test results must, so long as the number of stations is adequate, be attributed to lack of accuracy in those coefficients. Thus the main problem facing the structural engineer is to find a method that is adequate yet simple for deriving his coefficients—a point that is amplified in the Appendix.

Various methods are at present being tried by aircraft firms in this country, some of them not very different from the method used in the example I have taken, the basic notion of which is due to S. Levy[3].

For obtaining the influence coefficients of heavy-skinned thin wings, with a multiplicity of spars and ribs, the new method described in Part II of the Appendix appears very promising. Since however, it only occurred to me in the course of preparing this lecture, it has not yet been tried out. Like Levy's method it depends on deriving the force reactions due to a hypothetical set of displacements but, in contrast to that method, it takes adequate account of the continuity of the structure. The proposed method is also, I think, better suited to machine operation, in that the reaction at each station can be written down at once in terms of the deflections of the cluster of stations of which it forms the centre.

22. Time Saved By Machine

I need hardly expatiate on the time saved by the machine, as most engineers have read of the almost miraculous swiftness with which it can perform elementary operations on large numbers. A few figures directly related to the particular matrix operations already described may, however, be of interest. They are mostly based on a brief note[4] by M. Woodger relating to the performance of the A.C.E. Pilot Model at the National Physical Laboratory.

(*i*) *Individual members in structure*

As explained earlier, members such as spars, ribs, fuselage and fin are treated as beams for which influence coefficients must be calculated on anything up to about 10 stations. An example showing how to use the machine for a beam with 9 stations has recently been given by S. Levy[5] (using the S.E.A.C. machine at the National Bureau of Standards, Washington D.C.). After mentioning, in passing, that the machine takes only some 3 millisec. to multiply or divide two 13-digit numbers, he shows how, once the machine is programmed for the job, it is only necessary for the operator to specify the distance of each station from the root and the bending stiffness EI of the beam at each station—a few minutes' work. With this as a basis, the machine gives the whole set of 81 coefficients (each to 7 significant figures) as a 9-sided square matrix in about 3 min.

The parallel operation of computing (and checking) the coefficients by the use of a conventional desk machine took two days.

(*ii*) *Inversion of stiffness matrix for whole structure*

As already shown, the stiffness matrix has to be inverted to give the flexibility matrix preparatory to the iteration process. The machine times for the A.C.E. are as follows:—

(*a*) Using fast programme, available up to a 13-sided matrix ..... few minutes
(*b*) 16-sided matrix ..... 30 minutes
(*c*) 50-sided matrix ..... 5 hours.

These times may be compared with the following times achieved with a modern desk machine:—

(*d*) 6-sided matrix ..... 1½ days
(*e*) 13-sided matrix ..... 11 days
(*f*) 50-sided matrix ..... 2½ years

the time being roughly proportional to something higher than the cube of the number of elements per side of the matrix.

(*iii*) *Matrix multiplication*

As indicated in equation (47*a*), multiplication follows inversion and the machine times for the multiplication of two n-sided square matrices are as follows:—

For $n = 14$ — 1 minute
15 — 1½ minutes
31 — 8 minutes

(*iv*) *Iteration for frequencies and modes*

It is in the iteration process that the machine really comes into its own, for this is pure repetition work. To refresh the memory I will indicate again what is involved by means of the following symbolical equation.

$$\begin{bmatrix} g_{1,1}g_{1,2}g_{1,3}\cdots\cdots g_{1,50} \\ g_{2,1}g_{2,2}g_{2,3}\cdots\cdots g_{2,50} \\ \cdots\cdots\cdots\cdots \\ \\ \\ \cdots\cdots\cdots\cdots \\ g_{50,1}g_{50,2}g_{50,3}\cdots\cdots g_{50,50} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \cdots \\ \\ \\ \cdots \\ A_{50} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ \\ \\ \cdots \\ C_{50} \end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

It will be recalled that a single iteration consists in pre-multiplying the A "column" matrix by the 50-sided square matrix to give a new column matrix C. To obtain C_1 alone requires 50 multiplications of a couple of 7 or 8-digit numbers (decimal notation) and their subsequent addition, since

$$C_1 = g_{1,1}A_1 + g_{1,2}A_2 + \ldots\ldots + g_{1,50}A_{50}. \quad . \quad (64)$$

One single iteration therefore demands 2,500 such multiplications and anything from 20 to 30 iterations may be necessary before the A column repeats itself, which signals the derivation of only one of the natural frequencies together with its associated mode. With 50 stations it is possible to obtain something like the first twelve frequencies without more than two per cent. error, and the A.C.E. machine at the N.P.L. can turn out all twelve at the rate of about one per hour. To do the same work by means of a modern desk machine would be impracticable, as the time required would be of the order of *ten years*!

The figures just quoted indicate clearly the revolutionary character of the new factor that has entered the field of engineering calculations.

23. Static Stressing of Structure as By-product

Because the static elastic properties of the structure must first be obtained (either as a stiffness or a flexibility matrix) before the dynamical matrix can be constructed, we have, in effect, stressed the structure in the process of deriving its dynamical characteristics. Indeed, for multicell structures, such as the delta wing here taken as an example, a better method of stressing than that of deriving influence or stiffness coefficients for a fairly large number of stations distributed over the structure can hardly be imagined.

Looking back at the successive steps outlined here for deriving the dynamical characteristics of the delta wing, what appeared at the time as a necessary evil appears now as a positive advantage. I refer to the fact that it was found necessary to obtain the stiffness matrices for spars, ribs and fuselage, not directly but by the inversion of the corresponding flexibility matrices. It happens now that both types are necessary to the static stressing problem. For in this problem we are given the applied (aerodynamic) forces and have to calculate the stresses they induce.

To do this the first step is to represent, as nearly as possible, the continuous aerodynamic forces by a number of discrete forces, one at each station. The flexibility matrix (for the whole structure) then enables us to write down the deflections everywhere in terms of these forces. If now we require the stresses in (say) a particular spar, whose deflections are now known, we refer to the stiffness matrix for that individual spar, which at once gives the set of forces under which it is in equilibrium. The corresponding stresses are then easily calculated.

24. Precise Definition of Aerodynamic Forces

It is to be pointed out that this method allows a degree of refinement in the specification of the applied loads never previously approached, in that it can take full cognisance of the *chordwise* distribution of aerodynamic forces. For wings of moderate or large aspect ratio this is of little or no importance, but for stubby wings and deltas it is obviously much more realistic than the usual assumption of local concentration at the quarter-chord combined with a moment about the quarter-chord axis. The greater accuracy with which wing distortion can thus be calculated will also provide a more reliable basis for estimating such performance factors as aileron power, rate of roll, and longitudinal stability.

It is, I think, a good guess that all stressing of complicated structures will in the future be done by this method of discrete stations—apart, of course, from the stressing of detail parts such as spar joints and local discontinuities. If this is a correct view, then the high-speed digital computer is going to play, with the passage of time, a progressively more impressive role in aeronautics.

25. Some Final Remarks

The method outlined for deriving the influence coefficients for a delta wing is only presented as an example and is not necessarily the best. Some of its disabilities are discussed in Part I of the Appendix, in Part II of which is presented in outline the new method already mentioned. This, I would repeat, would seem to be peculiarly applicable to thin heavy-skinned wings of any shape, and to have the advantage of making the process of deriving the stiffness coefficients so systematic as to offer the maximum scope for exploiting the powers of the machine.

I had intended to show a comparison between calculated frequencies and modes and those subsequently found by resonance tests on the actual aircraft. I have not done so because, in the very few cases in which both test and calculated figures were available, the structure on which the calculated values were based had suffered various design changes before the date of the tests, thus making a direct comparison hardly fair.

It is pleasing to record that, from what I have been able to gather, aircraft firms in this country are fully alive to the potentialities of the digital computer. Indeed, they are already making considerable use of the limited facilities at present available to them. They *are* limited, because a machine such as the pilot A.C.E. at the N.P.L. is in constant demand, and I can imagine nothing more frustrating for a firm than to have its data all prepared for the machine and then have to wait its turn in the queue—possibly for weeks—until the machine becomes available. The remedy is of course obvious, though somewhat costly in initial outlay.

APPENDIX

PART I

In Part I of this Appendix, I want to touch briefly on a few point that were skated over in the main text. As will be seen, the method of deriving the influence coefficients there taken as an example has some important drawbacks and this is the reason why an entirely new approach for deriving the influence coefficients for delta and low-aspect-ratio thin wings (swept or unswept) is outlined in Part II of this Appendix.

1. EFFECTIVE FLANGE OF SPARS AND RIBS

When the bending strength of the wing lies largely in the skin, and when there are a fair number of shear webs, a good assumption would seem to be to associate with each shear web half the skin between its two neighbouring webs. Once the skin becomes a major factor in the strength of a wing there is everything to be said for having plenty of shear webs, and particularly so for small aspect ratio and delta wings, for this obviates the loss of strength and stiffness from *shear-lag* at the wing root. If the ribs are closely pitched the skin should be shared between them in much the same way.

2. EFFECT OF POISSON'S RATIO

There is no obvious way of allowing for the effect of Poisson's ratio in the foregoing method described, yet this ratio must have a very real effect on curvature in the rib direction for deltas and other low-aspect-ratio wings. Poisson's ratio probably affects only slightly the spanwise curvature but it must have a dominating effect on the chordwise curvature, and hence on the camber of the aerofoil. The method described in Part II of the Appendix takes full account of this factor.

3. TORSION-BOX CONTRIBUTION TO STATION REACTIONS

If the spar and rib webs are assumed to have no shear deflections—not an unreasonable assumption for thin wings—the corner reactions set up by a rectangular torsion box can easily be written down in terms of the corner deflections.

Figure 10(*a*) represents a torsion box in which stations 1, 2, 3, 4 are situated at the four corners AA_1, BB_1, CC_1, DD_1 and the neutral surface of which is marked by the dotted rectangle *efgk*. If now the sides of the box are tilted while remaining rectangular (by virtue of rigidity in shear) the top and bottom surfaces of the box become strained in shear as shown in Fig. 10(*b*) and (*c*). For example, if the side AB_1 tilts through an angle γ_1 and the side DC_1 through γ_2, the top edges AB and DC will be displaced in their own direction by amounts $h\gamma_1/2$ and $h\gamma_2/2$ respectively, where h is the depth of the box. This means that in Fig. 10(*b*) the side DC shifts to $C'D'$ so inducing a shear strain α given by

$$\alpha=\frac{h(\gamma_2-\gamma_1)}{2b}. \qquad . \quad . \quad (65)$$

But γ_1 can be represented in terms of the *upward* deflections w_1 and w_2 as

$$\gamma_1=(w_1-w_2)/a. \qquad . \quad . \quad (66)$$

[The representation in this Appendix of deflections normal to the wing plane by w rather than y is to be explained by the desire to follow the conventional rotation used in plate theory, where xy is usually the plane of the plate and w is the displacement normal to the plate in the direction z.]

Similarly

$$\gamma_2=(w_4-w_3)/a. \qquad . \quad . \quad . \quad (67)$$

Thus
$$\alpha=\frac{h}{2ab}\{(w_4-w_3)-(w_1-w_2)\}. \qquad . \quad (68)$$

The corresponding shear angle β of Fig. 10(*c*) comes to the same value, so that the total shear strain

$$2\alpha=\frac{h}{ab}(w_2+w_4-w_1-w_3) \qquad . \quad (69)$$

and the shear force per unit length of *any* edge of any side is

$$S=2\alpha Gt, \qquad . \quad . \quad . \quad (70)$$

where t is the skin thickness.

This at once gives (since the top and bottom surfaces have equal and opposite shear strains) an upward vertical force over the corner edge CC_1, of station 3, of amount

$$2hS=\frac{2h^2Gt}{ab}(w_2+w_4-w_1-w_3) \qquad . \quad (71)$$

$$=\frac{4IG}{ab}(w_2+w_4-w_1-w_3),$$

since $h^2t/2$ is the I of the box section per unit width.

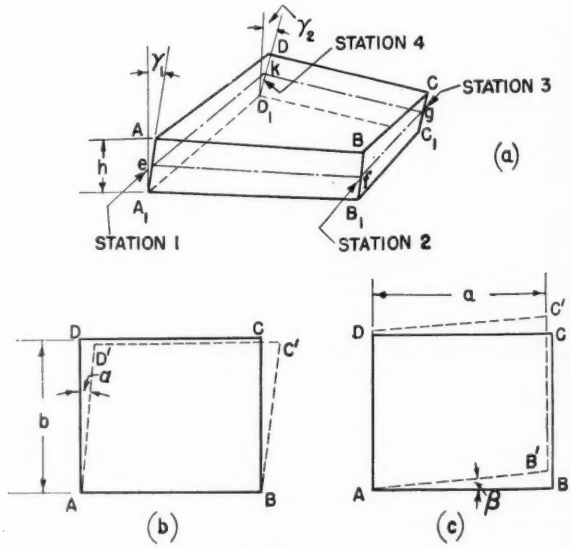


FIGURE 10.

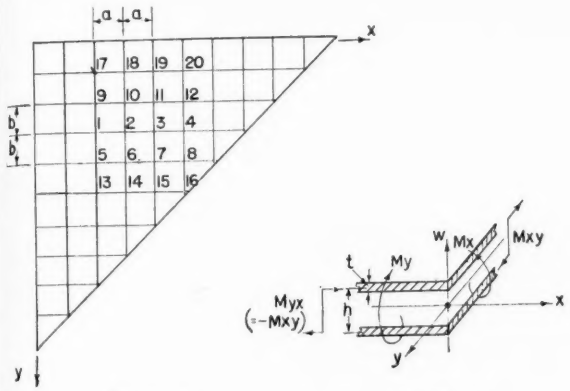


FIGURE 11.

FIGURE 12.

The upward force reaction at station 2 is therefore

$$F_2 = \frac{2h^2Gt}{ab}(w_2 + w_4 - w_1 - w_3). \quad . \quad (72)$$

It follows at once from equilibrium that

$$F_4 = F_2$$
$$F_3 = F_1 = -F_2. \quad . \quad . \quad . \quad (73)$$

These are the reactions due to the distortion of the single box cell *ABCD*, but the total reaction at any corner must include the three reactions caused by the distortion of the three other cells that meet at that corner. It follows that the reaction at a given station depends upon its own deflection and the deflections of the 8 stations immediately adjoining, as discussed more fully in Part II of this Appendix.

PART II

4. A SUGGESTED NEW METHOD OF TREATING HEAVY-SKINNED THIN WINGS

The method just described for taking account of skin shear is the same as that used for obtaining the twisting moments in plate theory. This suggests that, if plate theory is followed for obtaining the bending moments as well, a thin wing can be treated as a plate (solid or hollow) of variable thickness, thus allowing the familiar formulae of "small deflections" plate theory to be applied. The difficulty associated with Poisson's ratio then disappears and the whole procedure of deriving the influence coefficients of a thin wing becomes greatly simplified.

That the deflections (relative to a developable surface) are small enough to come within the limitations of "small deflections" plate theory can be shown. Taper in wing thickness raises no real difficulties in the case of thin wings—each elementary cell will of course be treated on the basis of an average depth of wing at that location. Boundary conditions are also easily allowed for, as will be shown below.

What the proposal amounts to is to replace the twin differential equations of plate theory[6]

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} + Q_y = 0 \quad . \quad . \quad (1A)$$

$$\frac{\partial M_{yx}}{\partial y} - \frac{\partial M_x}{\partial x} - Q_x = 0 \quad . \quad . \quad (2A)$$

(where the Q's are upward shear forces at the sides of the cell remote from the origin) by difference equations in which the elements of length dx and dy are replaced by the length and breadth of individual box cells. The same replacement allows the curvature of the wing, and hence the moments M_x, M_y and M_{xy}, to be expressed in terms of the deflections at box-cell corners. Thus, instead of the general equation for the bending of laterally loaded plates

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

we obtain a relation connecting the cell-box corner reactions with the 3rd differences of the deflection w.

It should be mentioned that the device of dividing a structure into a network of cells in order to convert the standard equations of equilibrium of the theory of elasticity into difference equations is precisely what is done in Southwell's[7] well known relaxation technique. Whereas, however, in the relaxation process the deformation of a structure by a given load is only obtained by the gradual successive liquidation of unbalanced forces, the machine gives the answer directly.

A brief outline of the method as applied to the kind of delta wing already considered is given here.

Suppose the delta wing of Fig. 11 to be divided up into a chessboard pattern of spars and ribs, of which a central group of intersections are given station numbers for reference. By analogy with plate theory[6], but reckoning w and z positive upwards, the *bending moments* M_x and M_y (positive in direction shown) per unit distance along y and x respectively are given by

$$M_x = \frac{Eh^2t}{2(1-\nu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right). \quad . \quad (3A)$$

$$M_y = \frac{Eh^2t}{2(1-\nu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad . \quad (4A)$$

where

$$h^2t/2 = I \text{ of section per unit width}, \quad . \quad (5A)$$

and ν = Poisson's Ratio.

The *twisting moment* M_{xy} (positive in the direction shown) is similarly given by

$$M_{xy} = -M_{yx} = 2GI\frac{\partial^2 w}{\partial x \partial y}. \quad . \quad (6A)$$

On using (3A), (4A) and (6A) in (1A) and (2A) we obtain the wing shears (per unit width) in the x and y directions as

$$Q_x = \left(\frac{-EI}{(1-\nu^2)}\right)\frac{\partial}{\partial x}\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right\} \quad . \quad (7A)$$

$$Q_y = \left(\frac{-EI}{(1-\nu^2)}\right)\frac{\partial}{\partial y}\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right\} \quad . \quad (8A)$$

where the quantity in the curly brackets is the sum of the curvatures in the x and y directions.

As an example, let us obtain the reaction at station

2. The average slope $i_{2,3}$ between stations 2 and 3 is given by

$$i_{2,3}=(w_3-w_2)/a$$

and similarly,

$$i_{1,2}=(w_2-w_1)/a.$$

The curvature, or rate of change of slope, is therefore

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_2=(i_{2,3}-i_{1,2})/a$$
$$=(w_1+w_3-2w_2)/a^2. \quad . \quad . \quad (9A)$$

Similarly $$\left(\frac{\partial^2 w}{\partial y^2}\right)_2=(w_{10}+w_6-2w_2)/b^2. \quad . \quad (10A)$$

To simplify the algebra let us assume that b and a are equal, although keeping them different would not make the treatment less straightforward.

The sum of the curvatures at station 2 (call it Z) is then

$$(Z)_2=(w_1+w_3+w_6+w_{10}-4w_2)/a^2. \quad . \quad (11A)$$

Similarly

$$(Z)_1=(w_0+w_2+w_5+w_9-4w_1)/a^2 \quad . \quad (12A)$$
$$(Z)_3=(w_2+w_4+w_7+w_{11}-4w_3)/a^2 \quad . \quad (13A)$$
$$(Z)_{10}=(w_9+w_{11}+w_2+w_{18}-4w_{10})/a^2 \quad . \quad (14A)$$
$$(Z)_6=(w_5+w_7+w_2+w_{14}-4w_6)/a^2. \quad . \quad (15A)$$

By (7A) the average shear (per unit width) between stations 1 and 2 is

$$(Q_x)_{1,2}=-B(Z_2-Z_1) \quad . \quad . \quad (16A)$$

and between 2 and 3, it is

$$(Q_x)_{2,3}=-B(Z_3-Z_2) \quad . \quad . \quad (17A)$$

where $$B=(EI)/(1-\nu^2), \quad . \quad . \quad (18A)$$

and I = section moment of inertia per unit width.

Similarly

$$(Q_y)_{10,2}=-B(Z_2-Z_{10})/a \quad . \quad . \quad (19A)$$
$$(Q_y)_{2,6}=-B(Z_6-Z_2)/a. \quad . \quad . \quad (20A)$$

It follows at once that the resultant upward reaction at station 2 (per unit width) is

$$R_2=(Q_x)_{2,3}-(Q_x)_{1,2}+(Q_y)_{2,6}-(Q_y)_{10,2}$$
$$=-B[(Z_3+Z_1-2Z_2)+(Z_6+Z_{10}-2Z_2)]$$
$$=-B[Z_1+Z_3+Z_{10}+Z_6-4Z_2]$$
$$=\frac{-B}{a^3}[20w_2-8(w_1+w_3+w_6+w_{10})+$$
$$+2(w_5+w_7+w_{11}+w_9)+(w_0+w_4+w_{18}+w_{14})]. \quad . \quad . \quad . \quad . \quad (21A)$$

To obtain the total reaction at station 2 we merely replace the moment of inertia B *per unit width* by the moment of inertia of a *width a* of wing section.

Thus the stations, in terms of whose deflections the reactions at station 2 is expressed, form a standard pattern that is shown in Fig. 13, where the inner eight stations surrounding the central station 2 form a square.

The form of the deflection function inside the square brackets of (21A) may be conveniently expressed in terms of this square as:—

20 times the central deflection
−8 times the sum of the 4 mid-side deflections
+2 times the sum of the 4 corner deflections
+1 times the sum of the 4 outer station deflections.

A major assumption, implicit in equations (3A), (4A), (6A), (7A) and (8A), is that shear deflections are negligibly small compared with bending deflections. For thin heavy-skinned wings, well supplied with shear-webs and ribs, it is a justifiable assumption. Moreover, once accepted, it is an assumption that allows us the further simplification of ignoring the precise geometry of the internal arrangement of ribs and shear-webs and of marking out the whole wing surface into a chess-board pattern of stations as in Fig. 11.

As already pointed out, to determine the reaction at every station in terms of the relevant deflections is to obtain a set of equations of type (39) and hence to obtain the stiffness matrix of (39*a*). The procedure thereafter follows the lines already discussed.

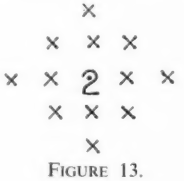


FIGURE 13.

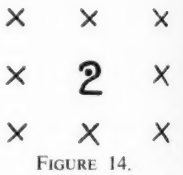


FIGURE 14.

BOUNDARY CONDITIONS

In this method of approach the boundary problem, always the main difficulty in analytical solutions, ceases to be a problem. For when the wing is held by a set of reactions in a given contour of deflections, the reaction at each station, whether on the boundary or not, is directly dependent upon and must be calculable from the relative deflections of the adjacent stations.

Suppose, for example, that all the structure to the right of station 2 in Fig. 11 disappears, so locating station 2 (and also stations 18, 10, 6 and 14) *on* the boundary. There is now no station 3 to define the slope between 2 and 3 which normally, in conjunction with the slope between 1 and 2, gives the x curvature at station 2. The fact that the y curvature is still explicit, together with the fact that the bending moment M_x in the x direction must be zero at the boundary, is, however, enough to enable the x curvature to be written down at once. For, by (3A), since M_x is zero,

$$\frac{\partial^2 w}{\partial x^2}=-\nu\frac{\partial^2 w}{\partial y^2} \quad . \quad . \quad . \quad . \quad (22A)$$

which gives the x curvature, and hence the sum of the curvatures Z_2, in terms of the known y curvature. The values Z_{10} and Z_6 at stations 10 and 6 are found in the same way. The reaction at station 2 then follows from equations (19A), (20A) and (21A), as before.

A procedure of this kind can be applied to all free edges of the wing so long as the true boundary is

represented, as it can be with little loss of accuracy, by a scalloped boundary. For the wing root the boundary conditions are equally straightforward and no real difficulty arises.

MACHINE COMPUTATION

The procedure just described for deriving the reactions at successive stations appears systematic enough to be readily programmed for computation by the machine. To obtain the stiffness coefficients for any particular wing the distinctive data to be fed into the machine would then consist of

(1) The spacing of the stations

(2) The cross-sectional moments of inertia at each station

(3) The wing plan form.

The stiffness matrix thus found by the machine is then readily inverted to give the flexibility matrix, *i.e.* the whole set of influence coefficients.

These not only form the basis for the iteration processes that are to give the natural frequencies and modes of the structure; they also constitute the essential structural data for solving aeroelastic problems of a non-oscillatory type, such as the calculation of aileron power, aileron reversal and wing divergence.

Not least important is the fact that, to derive the complete set of influence coefficients, is to solve the wing stressing problems. This follows from the fact that the bending moments and shears at any station on the wing can be written down for any given distributed loading.

Other important advantages of this method are:—

(*i*) The wing deflections and stresses derived by its use take full account of the actual distribution of aerodynamic loads, instead of relying on the crude assumption of a line loading along the quarter-chord and a couple about that line, which is the universally accepted current practice—a practice good enough for wings of moderate aspect ratio but not for stubby wings or deltas.

(*ii*) It automatically eliminates the twin bogies that normally complicate the problems of deriving the stresses and deflections of a wing—namely shear lag and the warping of cross sections.

(*iii*) It takes account of the flexibility of the fuselage as a factor affecting both the stresses and the deflections of a wing. This is a much needed corrective for the all-too prevalent tendency to simplify wing structural problems by assuming a completely encastré root fixing.

NOVELTY OF METHOD

There is nothing novel about using elementary plate theory for the structural analysis of thin wings—at least of solid thin wings.

Reissner and Stein have used it in recent N.A.C.A. reports[9, 10] for solid thin wings of simple cross section. Fung[11] has used it—also for a solid plate—in conjunction with Southwell's Relaxation Method.

Any novelty the method may have therefore lies in the notion of combining plate theory with machine computation of the influence coefficients.

Without the machine the method has no interest, but with the machine it may well have far-reaching possibilities.

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Discussion

DR. P. B. WALKER (Royal Aircraft Establishment, *Fellow*): In his opinion, flutter and aeroelastic work had reached a rather dangerous age. There now existed mature specialists in the flutter field, or at any rate people who had done practically nothing else throughout the whole of their working lives. Those of them who knew Dr. Williams would appreciate that while he was not without distinction in the aeroelastic and flutter field he had, as the Chairman had shown, roamed about the whole of the structural field. They had therefore been presented with a picture of aeroelasticity related to the digital computer as seen by someone who, although a specialist, was not limited by specialisation.

He felt that the matrix and digital computation problem had been made clearer than it ever was before, although he was not prepared to admit that it was quite so simple as Dr. Williams would like them to believe.

There was one incidental point that had impressed

him about the talk that night. That was that flutter was very often regarded as primarily an aerodynamic problem. It was obvious, however, that the problem of flutter was as much an inertia and an elastic problem as an aerodynamic one, and he felt that that was brought out very clearly in the lecture.

Dr. Williams mentioned model experiments as a kind of rival to the digital computer. Unfortunately he closed that paragraph by stating that he was not going to say any more about models. Suppose they had all the data with which to feed the digital computer, which he thought was a fair assumption, why did they need to make a model? There might be some models which could deduce more information than they put into them. As a simple analogy, they might use a flexible strip to draw a curve on a drawing board and make it fit six points. It would then automatically give a fairly reasonable estimate of the slope at each point and the curvature, because it was a physical thing. Did this mean that a model was really of value, apart from educational value, when the digital machine was available? Was there any purpose in making a model, which would take time and be expensive, if they need only to put the data into the digital machine to get the answer?

DR. S. NEUMARK (Royal Aircraft Establishment, *Associate Fellow*): The original title of Dr. Williams' lecture was "Modern Developments of Aeroelastic Theory"; the final one was "Recent Developments in the Structural Approach to Aeroelastic Problems." The paper dealt with most recent developments in the structural aspect, and he would like to congratulate Dr. Williams especially on having narrowed down the subject so that they were really able to learn something about it. They had been used to so many lectures to the Royal Aeronautical Society giving general reviews of immense subjects, and although such reviews were, of course, of great importance, many members such as himself had often left the meetings feeling rather humiliated by their own limitations. Lectures such as that by Dr. Williams, however, were very much to his own liking and possibly to that of many other members. Could they please have more such lectures?

He thought perhaps this lecture on aeroelasticity could well be followed by another which would present the aerodynamic side, and then perhaps somebody would try to summarise and bring the whole lot together, to give a general outlook on this branch of aeronautical science. He thought the last of those lectures would be most difficult, and probably it would be somewhat difficult to find a lecturer. The lecture would have to deal with four groups of problems in the order of increasing difficulty, namely

(*i*) Problems independent of mass distribution, for which a representative example was that of aileron reversal,

(*ii*) Basically structural problems, affected by aerodynamics, such as wing and control-surface-flutter.

(*iii*) Basically aerodynamic (or flight-dynamical) problems, affected by elastic distortion—such as dynamic stability, manoeuvre margin and so on.

(*iv*) The highest order problems, dealing with coupled disturbances of the entire aircraft and its internal structure, where aerodynamic and structural aspects were completely blended together.

Coming back to Dr. Williams' lecture, what was most striking to him was the story of the revolutionary change brought about by the introduction of the digital computer. He agreed with everything Dr. Williams had said about that, except an emphasis on one point. He did not wish to be misunderstood. It meant a lot if they could calculate the effects of elastic distortion on a particular aircraft in a week or two while the old methods would require a year. From the general research point of view, however, it was rather different. He thought there was a certain danger that newcomers into research would substitute a machine for thinking. If people could feed their data into machines and get the right answers in a short time, that was due to the fact that much thought and much suffering of pain and tears had been devoted to the design and development of such machines. He thought there were signs already that newcomers to the research field felt so impressed by the extreme efficiency of the new machines that it would be good enough for them just to learn to use these monsters, which would provide the answers, and then they would be free from further thinking. That was not a joke, but a very serious matter, and the danger was there. He did not know what to propose as an antidote. Perhaps it would be a good thing not to allow young people to sign (as authors) papers containing exclusively or mainly results obtained by the use of digital computers. This would not apply to the older men, who were often supposed not to need to think!

The entire problem of digital computers versus the older methods was very simply illustrated in the case of the pendulum. If you had a flexible chain which oscillated as a pendulum, the simplest course was to consider it as one particle on a string and this would give a single mode and frequency being a first rough approximation to the basic mode of the chain. The second approach would consider the oscillating system as consisting of two particles, and then they would have two modes and frequencies, approximating the two true lowest frequencies and so on. When the problem was first put, the people concerned had to rely merely on hard thinking, and the final solution of the problem by Bessel functions, giving the entire infinity of modes, could be written practically in one line. Then one might consider any initial shape of the chain, and describe its oscillations with any accuracy required. If the scientists of 150 years ago had had the aids now available, they would have been able to calculate 8 or 10 modes of a chain, but they would perhaps have no Bessel functions, nor hundreds of other similarly valuable tools.

The lecturer stated that solution by iteration led directly to the lowest frequency in the case of flexibility matrix or to the highest frequency in that of stiffness matrix. He knew it was so in practice, and he wondered whether there was a definite proof of this and where it was to be found. If they took, accidentally or deliberately, as a first approximation, the amplitude ratio corresponding (say) to mode No. 3, the result would

have to be the frequency of mode 3. If they happened to start with an approximation very near to the solution of No. 3, should they not obtain the exact solution?

Another troublesome question seemed to be that of mass distribution. When one had (say) 40 or 50 stations on the wing surface one had to attribute a certain part of the mass to each station. How should this distribution be effected without causing a serious mistake? Would two independent workers not come to considerably different results from the same set of data?

Some time ago, both aerodynamic and elastic aspects seemed fairly simple, the former being solved on the assumption of lifting line and quasi-steady flow, the latter on the assumption of flexural axis. Since that time, they had to introduce lifting surfaces and unsteady flow on one side and replace the old good flexural axis by a less digestible flexibility matrix. However, one welcome effort towards a new simplicity was made recently, at least on the elastic side, by Broadbent, leading to his Q-locus as a generalisation of flexural axis. He wondered what was the mutual relationship between this and the approach by full matrices.

Finally, structural damping had been neglected in Dr. Williams' paper, so that they had to consider undamped oscillations. That was certainly good enough where elasticity dominated. But it might not be like that in problems where the interest focused on damping, and they knew from bitter experience in purely aerodynamic problems that, if they had several degrees of freedom, and one high-frequency mode was too well damped, then the low frequency oscillations might exhibit dangerous negative damping. Neglecting the structural damping, might not always mean being on the conservative side. Was the flexibility matrix to be supplemented by an even more formidable "structural damping matrix"?

H. H. GARDNER (Vickers-Armstrongs Ltd., *Assoc. Fellow*): He was very impressed by the final point on which the last speaker touched. They knew Dr. Williams' breadth in the field of structures and were very conscious that he was one of the few people who had spent much time on aeroelasticity. He had highlighted one particular area in the field of aeroelasticity tonight, which suggested that the digital computer and the use of the matrix was undoubtedly one of the most important developments in the new field of flutter. Much thought went into a paper of this type, and he could only assume that Dr. Williams had crystallised that as the most important feature of flutter work at the present moment. Work on aeroelasticity had been going on for many years, and undoubtedly the digital computer opened up a wider field. The last speaker asked for another lecture on the analogue computer and then for someone to sum up the relative merits of the two. They should not pass this by without asking Dr. Williams what his view was at this stage of the digital computer and of the value of the work using matrices, compared with the work done in flutter simulation. A brief summing up of the relative places of the methods in the field of aeroelasticity would be interesting.

The paper provided a fine example of the mathematical approach, and they owed Dr. Williams a great deal for having put this forward in such a way that people in the various sections of the Industry would be able to apply it to the work they were doing. They were more and more dependent on mathematicians, a matter which he thought was inevitable and undoubtedly would continue. Nevertheless, he felt there was the danger that if they allowed mathematicians to go too far without proper control they might run into more trouble through the purely theoretical approach than they needed to do. After all, that was the basis of calculation as regards the actual modes of flutter. The actual treatment of an aeroplane before flight, based on those calculations, was sometimes a very difficult thing to decide, and very often many difficult decisions had to be made as the result of those calculations. He thought, therefore, that to allow mathematicians freedom of range in this field could be done only under engineering control. He would suggest that in the Industry they had to watch the effect of mathematicians on the field of flutter. Young mathematicians employed in this work would produce a large amount of data and the machine would be used more and more, the digestion of the variety of results would take longer, and there was danger unless there was effective control.

E. G. BROADBENT (Royal Aircraft Establishment, *Fellow*): Dr. Neumark had referred to a paper of his dealing with aileron reversal and the use of flexibility coefficients. He did not think he need say much about it, except that Dr. Williams' method would, of course, provide all the data that was required by his method for calculating the control reversal speeds.

Another point which Dr. Neumark made concerned the structural damping in different modes. Dr. Neumark pointed out that it had often been assumed that, if the flutter speeds were calculated for no assumed structural damping, a conservative assumption was being made. They had realised that that was not necessarily so and that structural damping might in some cases lead to a slight reduction of flutter speed as compared with no structural damping. Now that they had those elaborate electronic machines they had no need to make such assumptions.

He would also agree wholeheartedly with Dr. Neumark that it was very necessary to think what one was doing as well as being able to use the machine to the fullest availability. It was so easy to draw up very large matrix numbers even through various machines and get an answer which said there was a margin of 25 per cent. on the design diving speed with regard to flutter and various other aeroelastic problems, and to say that everything in the garden was lovely. But if one were not aware of what was causing various parts of that calculation to have different effects, something quite important might be missed; even taking a large number of degrees of freedom in a calculation, it did not ensure that they were getting the exact solution of the physical problem.

SIR HAROLD ROXBEE COX (Chief Scientist, Ministry of Fuel and Power, *Fellow*): It seemed to him that with

these devices, capable of such a wealth of accurate computation, it was important that the data fed in was of corresponding accuracy. In other words, while the machine itself might be capable of performing wonders in the provision of information from given data—in this case, as he understood it, the influence coefficients—it would seem to be extremely important to take into account all the factors involved in the quantitative definition of those coefficients. Could they be found with sufficient accuracy by calculation, even if the machine itself were used? Or were they best obtained by experiment with a model or a full scale structure?

He could not help recalling, when Dr. Williams was talking, the remark they used to hear when relativity was new that, if a thousand monkeys were given typewriters, if they typed for long enough they would eventually produce the whole of Shakespeare's works. If anybody should desire to test the accuracy of that prophecy he now had the means to hand. Nevertheless, he felt there might be a factor which would make even less extraordinary exercises impracticable. He might be exposing himself to an accusation of abysmal ignorance, but at one stage during his talk Dr. Williams did imply that before such and such a calculation could be done the machine "had to be programmed." He had heard those words before uttered with somewhat sinister import. He was at a meeting where it was suggested that a machine could in fact do the job of the air traffic controller; but to ensure that if the right kind of information were fed into the machine it would produce the right answers required elaborate programming, or preparation for the task. Was Dr. Williams hiding something from them, or was the programming in fact a comparatively small part of the work?

G. R. EDWARDS (Chairman): Now that digital and analogue computers were here, he wondered how they had managed to get by without them, in the solving of problems which under the conditions obtaining earlier would have required ten years of work.

In the United States he was struck by the great array of equipment used. He thought it was used to excess.

Dr. Williams knew how to use these particular machines for his part of the job, which they had discussed; and there was no doubt at all that those machines would be an aid to development in aviation. He was sure they would all wish him, on their behalf and his own, to express to Dr. Williams their very deep thanks for the work that had gone into the preparation of the paper, and to say how grateful they were.

A. S. TAYLOR (Royal Aircraft Establishment, *Assoc. Fellow*), *contributed*: The development of the subject of aeroelasticity had tended to take the form of alternate steps forward on the structural and aerodynamic sides respectively, so that at one stage, the relevant aerodynamic theory would be lagging behind the structural theory; at another stage, the positions would be reversed.

Dr. Williams, representing the structural school, had now taken a mighty leap forward, from a slightly rearward position, to put his "side" ahead of the aerodynamicists. But clearly, the aerodynamicists might quickly regain the lost ground if they also enlisted the aid of the machine, since the problem of estimating theoretical spanwise and chordwise loadings to any degree of accuracy was already solved in principle.

As Dr. Williams had observed, his method of specifying the structural properties of a wing allowed them to take full cognisance of the *chordwise* distribution of aerodynamic forces, and of course at the same time, they would be able to investigate elastic camber effects which might assume major importance for delta and other small aspect ratio wings. In saying this, however, they presupposed the availability of readily applicable methods of calculating spanwise and chordwise loadings for wings of arbitrary shape with given spanwise and chordwise distributions of local incidence.

Now while, as already stated, that generalised problem had been solved in principle, in the methods of such people as Multhopp, and whereas the detailed development of such methods had generally been taken far enough to give an adequate representation of the spanwise distribution, the mathematical description of the chordwise loadings implicit in the methods was not usually sufficiently detailed to do more than ensure that the local aerodynamic centres were determined with reasonable accuracy. At the same stage of development, those methods could not make direct use of the local incidences at more than two (or possibly three) chordwise locations.

Ideally, if they were approaching one of the static aeroelastic problems from the structural side, and wanted to make fullest use of Dr. Williams' ideas, they would like to be able to calculate directly a set of loads which acted at the points for which the flexibility coefficients were defined—let them call them the structural pivotal points—those loads being equivalent to the distributed aerodynamic loading corresponding to a set of local incidences either at the structural pivotal points, or at a closely related set of points (*e.g.* points midway between adjacent pivotal points of the same chordwise section).

Unfortunately, from the aerodynamic point of view, the structural pivotal points would generally constitute an arbitrary set, unrelated to any set of pivotal points which one would normally select in dealing with a purely aerodynamic problem. The point thus arose as to whether they could, without too much inconvenience, adapt their lifting surface calculations to a foreign system of pivotal points, or whether Dr. Williams could readily transform his set of influence coefficients, worked out for the most obvious system of *structural* pivotal points, to an equivalent set of coefficients related to a suitable system of *aerodynamic* pivotal points.

Whichever course they attempted to follow, the computations would be long and involved, and would demand the use of the machine. Two questions which they must then seriously consider were:—

(*i*) Did the extra accuracy they could achieve justify the degree of complexity introduced into the computations, even allowing for the fact that the machine could perform in a few hours, calculations that would occupy the human computer for ten years?

(*ii*) If the machine approach to these and similar problems became popular, were there going to be enough machines and operators? Would they have to wait in the queue for the ten years that would have sufficed to do the job by hand?

PROFESSOR W. J. DUNCAN (University of Glasgow, *Fellow*), *contributed*: He was very pleased to see that Dr. Williams had become a convert to matrix methods in their application to structural and vibration problems and was sure that Dr. Williams' paper would do much to increase the use and appreciation of these very valuable methods. Might he be pardoned for pointing out that the book, "Elementary Matrices" (Ref. 2 of Dr. Williams' paper) was published in 1938, not in 1946, and thus antedated "Mechanical Methods in Engineering," which indeed contained a reference to "Elementary Matrices." Since Dr. Williams began the title of his paper with the words "Recent Developments" it might be pointed out that the dynamical matrix was introduced by Duncan and Collar in 1934, *i.e.* twenty years ago. He appended three references which were relative to this aspect of the matter.

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R. K. LIVESLEY (Manchester University, Computing Machine Laboratory), *contributed*: As his work had been concerned mainly with building frameworks, he did not know whether the matrix approach put forward by Kron[1][2] would be equally suitable for aircraft structures. Whatever method was adopted, however, he agreed entirely with the author that it was easier to set up the stiffness matrix than the flexibility matrix. The stiffness matrix of the complete structure was the sum of the stiffness matrix of individual members[2], while no such simple addition rule could be used to combine the flexibility matrices.

Once the stiffness matrix and the various inertia forces had been calculated, the remaining problems were essentially computational. The basic equation (10) might be written

$$([a]-[m]\,w^2)\,A=0 \qquad . \quad . \quad (1)$$

where $[a]$ and $[m]$ were the stiffness and inertia matrices —$[m]$ normally being diagonal. The author's reduction to the form given in equation (11*b*) resulted in an asymmetric, "dynamical matrix." The following approach retained the symmetry, which might have some advantages in the subsequent computation.

They wrote $[a]=[l]\,[l']$, where $[l]$ was a triangular matrix and $[l']$ was its transpose. (This was the first step in the Choleski method of matrix inversion.) It was easy to invert $[l]$, and they also had $[l']^{-1}=[l^{-1}]'$, so that they could re-write equation (1), above, as

$$([l^{-1}]\,[a]\,[l^{-1}]'-[l^{-1}]\,[m]\,[l^{-1}]'\,w^2)\,A=0, \qquad (2)$$

whence

$$([I]-[b]\,w^2)\,A=0, \quad \text{or} \quad ([b]-\frac{1}{w^2}[I])\,A=0, \quad . \qquad (3)$$

where $[I]$ was the unit matrix and $[b]$ was now symmetric. He mentioned this point as it was often easier to extract the latent roots of symmetric matrices.

The author considered the iterative method of finding the natural frequencies. That was not the only one available, and it was by no means certain that it was the best. The Lanczos process[3] had been coded for the Manchester computer by R. A. Brooker and F. H. Sumner[4], and had proved its worth on many large matrices, some with sets of repeated roots. Its advantages could be estimated from the following details.

26-sided matrix with one root repeated 6 times

Formation of characteristic equation 10 min. (approx.)
Computation of roots 10 min. (approx.)

The inversion time for such a matrix would be about 7 minutes.

The Lanczos method had the drawback of requiring more storage space than the method described by the author. It was thus more suited to a machine like the one at Manchester, which had a large magnetic store. The A.C.E. was basically the faster machine, and it was this large storage (which could retain all the elements of a 50-sided matrix at one instant) which was partly responsible for the differences in the times quoted.

A more important advantage of a large-store machine, however, was the fact that it became easier for the user to conceive of a programme as a unified whole. As the author said, the machine should be used for the initial setting up of the matrix coefficients as well as for the later manipulation. With a large store available, a programme could be developed in which input was confined to drawing-board data, while output was restricted to the final natural frequencies and normal modes. All the operations of matrix assembly and manipulation would then be arranged to take place, "behind the scenes," and such a programme could be used by designers with no knowledge of matrices at all. It seemed to him to be one of the attractions of a digital computer that a programme based on complex mathematics might be used confidently by a non-mathematician. The programme for structural analysis developed on the Manchester machine had in fact been used in this way on several occasions.

Although no expert on a desk machine, he thought that he could invert a 6-sided matrix in less than $1\frac{1}{2}$ days.

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DR. E. H. BATEMAN, *contributed*: Whether or not Bessel would have formulated his differential equation if the machine had been available in 1824, the fact remained that Bessel's functions had only been brought into full use as a result of the tabulation of a comprehensive range of functions by the electronic digital computer[1]. For, as Sir Richard Southwell had remarked in delivering his Hawksley lecture[2] in the same hall less than three months ago, an elegant transcendental solution was almost valueless in comparison with the complete resolution of a problem by numerical methods such as relaxation or machine computation. Since, although Sir Richard confined his observations to relaxational methods, there could be little doubt but that the machine would before long be made to solve any problem amenable to a numerical solution.

He had recently seen in the Harvard computing laboratory an electronic programming machine, of which the executive unit was a large manual with keys for setting up the four basic arithmetic operations for simple and complex numbers, and with keys for square roots, logarithms, exponential, trigonometrical and special functions, so that presumably before long, any type of programming, including that for Dr. Williams' equations, would very soon become fully automatic.

It seemed inevitable that when a user was paying a large rent for the machine ($12,000 a month in some United States aircraft design offices), there would be a tendency to keep it working as hard as possible, whether the results coming out were useful or not. The same thing had happened with other items of mechanical equipment, cameras, and recorders of all kinds. However, experience in operation was being continually accumulated, and this would enable worthwhile problems to be solved when the time came.

The author had shown how one of those problems could be solved by the machine, and although he had referred to it as an *ad hoc* solution it was, in fact, a characteristic or "eigen" solution for the particular structure considered. This was an important step forward towards the complete solution of the aeroelastic problem, in comparison with any of the iterative methods which had been proposed for structural problems.

The existence of the machine which was solely designed to reduce manual labour in computation, would by no means reduce the amount or quality of the mental effort required in aircraft stress offices, for if the machine were unintelligently used it might take far longer to solve a problem than an engineer with a pencil and postcard. The classical example of mis-application of the machine was that of the Manchester machine which, when programmed for the detection of prime numbers, took an hour or two to find the highest factor (*i.e.* the half) of a large even number. In the evaluation of latent roots—the author's main problem—convergence might be very slow if a pair of roots had equal or nearly equal moduli, but the difficulty was easily overcome by a slight transformation of the matrix followed by a rectification of the results; no doubt any difficulty arising from a critical value of the first trial solution could be even more easily overcome.

The author had referred to his work as a new approach and this put his contribution in the proper perspective, for the example he had given was surely only one of many problems which the machine could be tuned to solve for aeronautical engineers.

REFERENCES
1. Annals of the Computation Laboratory of Harvard University, Vols. III to XIV, 1946, *seq.*
2. R. V. SOUTHWELL (1954). Relaxation Methods: A Retrospect. *Journal I. Mech. E.*, Vol. I, p. 61, 1954.

N. P. SHEVLOFF (Group Leader, Flutter and Vibration, Short Brothers and Harland Ltd.), *contributed*: While congratulating Dr. Williams on his admirable commentary on theoretical development and future trends in the aeroelastic field, he would like to challenge him on one point in his paper. On page 413 Dr. Williams considered the boundary conditions of a symmetric "free-free" vibration and obtained equation (47). He then stated that equation (48) was obtained by the inversion of the L.H.S. matrix of equation (47) by the machine. This was a completely unnecessary step, as by alternate evaluation of the boundary conditions, the inverted matrix in equation 48 could be set up as a simple matrix multiplication product, involving the inversion of a 2×2 matrix only.

While agreeing that the machine time involved for the boundary condition inversion shown by Dr. Williams was comparatively small, the machine time at present available to Industry was limited by the small number of computers in the country, but, in addition, the avoidance of this inversion would, based on information in Ref. 4, save a sum of £13 to £80 for problems of the order 20 to 50 freedoms. As the total expenditure on a normal mode solution by machine was of the order £100 to £200, this was a worthwhile saving.

An alternative substitution for boundary conditions was included in work for general cases by Mr. T. J. Reid of Short Brothers and himself published in the December 1953 issue of *Aircraft Engineering*.

Dr. Williams' Reply

DR. WALKER: Dr. Walker was concerned about the precise place of structural models in relation to the digital computer, and was, he said, disappointed that he dismissed them rather summarily in a single paragraph at the beginning of the lecture. He did so, first because the technique of obtaining information via models could not be regarded as a recent development, and secondly because he wanted to concentrate on machine computation as the main theme. He did emphasise however that the one approach should be regarded as supplementing, rather than competing with, the other.

Dr. Walker made the further point that, since the essential data required to make the model was the same as that fed into the machine, it should never be necessary to make a model. The answer to that, surely, was that the one was an experimental approach and the other a theoretical approach. The fact that the machine helped in the calculation did not make the process less theoretical, and it was always reassuring to have theoretical results confirmed by experiment.

DR. NEUMARK: The suggestion that this lecture should be the first of a series dealing with various aspects of aeroelasticity was worth taking up.

He liked the point about the danger that, by resorting to the machine whenever a difficult problem had to be faced, young research workers would come to neglect possible alternative and more elegant analytical approaches.

He could not believe that danger to be real. Counting by fingers did not retard arithmetical progress, nor had the slide rule and the desk machines and the Relaxation Method impeded advances in analysis. And what, after all, were tables of Bessel's Functions except a set of numerical results designed to avoid tedious computation? As well condemn them for standing in the way of possibly more elegant methods of dealing with Bessel's equation!

As regards the proof that the method of iteration always gave the highest latent root first, that could be found in Refs. (1) and (2) in the printed lecture.

The question of what would happen if, in iterating for the 1st mode, one happened to choose for arbitrary starting mode the amplitude ratios corresponding to the 3rd mode, was he thought, clear. One would end up with the fundamental mode. This followed from the fact that the 3rd mode could not be quite exact in the first place and iteration would only make it diverge farther. The case was very different when the 1st and 2nd modes were deliberately excluded, for then the 3rd mode *was* the lowest remaining mode.

He foresaw no real difficulty about the distribution of mass. Two people would certainly choose slightly different discrete masses for the various stations, but not so different as materially to affect the final results.

As for the flexural axis, he feared the simplicity of approach it stood for had gone, never to return. Broadbent's Q locus was in no sense a substitute for the old flexure axis notion but rather a convenient datum for defining the deflection of the wing.

Dr. Neumark pointed out that structural damping, although small in itself, might, under certain conditions, have important effects when combined with aerodynamic damping. That might well be so, but the point was outside the field he had tried to cover, which was exclusively concerned with the structural side of aeroelastic problems. Structural damping did however come under that category and, small though it was, an estimate of its magnitude should be made for the benefit of the aeroelastician. Since it could not be calculated, any such estimate must be based on previous experience of similar structures.

MR. GARDNER: The important question of the relative merits and fields of usefulness of the digital computer "vis-a-vis" the differential analogue computer had been raised. As he saw it, these two mechanical aids were to a large extent mutually complementary, so that, although their fields of usefulness overlapped, they were none the less distinctive. The differential analogue computer, as its names implied, could only be used for solving differential equations, and was particularly useful, as in the flutter equations, for spotting the transition from stable to unstable motion brought about by varying the parameters of the problem. In this field it was supreme, but it was a somewhat restricted field compared with the vast area over which the digital computer had no rival. Every kind of calculation that could be reduced to a systematic set of operations—and there were few calculations that the competent programmer could not so reduce—was grist to the mill of the digital computer. Differential equations however, unless converted into difference equations, were outside its orbit except in so far as their solution could be reduced to the solution of a set of simple linear algebraic equations, as for example in the case of the undamped harmonic motions discussed in the lecture. On the other hand, the systematic calculation of the stiffness coefficients of a structure, and the inversion of the corresponding matrix, were beyond the province of the analogue computer.

One might summarise the position by saying that the structural approach discussed in the lecture was essentially the province of the digital computer. So also was the solution of non-oscillatory types of aeroelastic problems such as aileron-power and wing divergence. For investigating flutter, however, and all types of oscillatory instabilities, the analogue computer was without doubt the right instrument. This meant that when an analogue computer was alone available, the use of the large number of stations, necessary to the adoption of the structural approach he had described, was impracticable without access to a digital computer.

Mr. Gardner's warning of the danger of letting mathematicians, unless they had a sound knowledge of structures, loose on digital computers was very timely and he thoroughly endorsed it. What had to be remembered was that the correctness of the influence coefficients fed into the machine was a measure of the correctness of the answers, and when the machine itself was used to compute the influence coefficients, the instructions given to the machine must be informed by sound engineering knowledge.

MR. BROADBENT: Mr. Broadbent confirmed the point made by Dr. Neumark about structural damping, and any previous remarks again applied. He also called attention to one of the disabilities of machine computation in that, if it gave an unfavourable or unexpected answer, it was not easy to trace, among all the parameters involved, the true governing factor in that answer.

SIR HAROLD ROXBEE COX: With his usual happy knack for putting his finger on vital points, Sir Harold had raised two questions. How, he asked, were the influence coefficients to be found with sufficient accuracy, even with machine computation, and was there a possibility of obtaining them by direct measurement from a scale model? As he mentioned in commenting on Mr. Gardner's points, the accuracy of the coefficients depended on the soundness of the structural knowledge behind the machine computation: the machine did not enhance the accuracy—except in so far as it avoided arithmetical slips—but merely reduced the computing time. Another important point was that the necessary

structural knowledge should, as far as possible, be available to designing staffs as a standard crystalised procedure, requiring for its successful application only to be adapted to the particular aircraft design.

The use of a scale model, or any other kind of model, for obtaining influence coefficients experimentally was, in his view, to be deprecated. The very idea nullified the main advantage of the machine approach, which was to obtain a reliable idea of the characteristics of various alternative designs before commitment to any particular one. To succeed in this required that the calculations should be a matter of weeks and not months, and a suitable model would take months to make. The model, it seemed to him, had two functions to perform. First, by representing the inertia as well as the stiffness of the prototype, it provided a useful check in allowing a comparison to be made, not of its influence coefficients, but of its frequencies and modes of vibration with those obtained via the machine. Secondly, if it were an aerodynamic as well as an elastic-cum-inertia model, it could be used for experiments in the wind tunnel, where its aeroelastic behaviour could be observed directly.

Sir Harold's second point concerned the question of "programming" the work for the machine, a process, he said, that had been left unexplained. It had indeed! In the lecture he was careful to use the phrase "the machine and its acolytes" in referring to the working entity; the acolytes were intended to include the programmers. So far as he understood the matter, it was the programmer's job to split up the computation work so as to present it to the machine as a sequence of simple arithmetical operations that the machine could understand. It took, he was told, something like a couple of months of intensive training at "machine headquarters" to learn the art of programming.

MR. TAYLOR: Mr. Taylor had given an interesting picture of the aerodynamic situation in relation to aeroelastic problems, and had pointed out that the most convenient distribution of stations for defining the structural properties of a wing was unlikely to be the most convenient for defining its aerodynamic properties. Arising from this, Mr. Taylor wondered whether the set of influence coefficients worked out for the most obvious system of pivotal points (*i.e.* stations) could be transformed into an equivalent set more suited to the aerodynamic problem.

It could be said in answer that such a transformation offered no difficulty, and therefore the aerodynamic forces could be derived on the basis of a completely free choice of stations.

Mr. Taylor asked further whether the extra accuracy justified the extra complexity. He would say that everything depended on the importance of the extra accuracy. If a simpler (and rougher) approach were likely to give a false idea of the behaviour of the aircraft, the more accurate approach was well justified.

As to a possible scarcity of machines in the event of machine computation becoming popular, that might certainly be a temporary difficulty, but supply in the end usually overtook demand.

PROFESSOR DUNCAN: He was glad that Professor Duncan thought the paper would help to popularise the matrix approach to engineering problems. As for the date quoted in the reference to the book "Elementary Matrices" he was concerned less with the historical aspect than with indicating the latest impression for the benefit of future students of the book. Even so, his position was apparently still vulnerable, for the latest impression, complete with corrigenda, was dated 1950.

As for the *Phil. Mag.* article of 1934, he studied that with profit early on, but omitted it as being substantially included in the later book.

MR. LIVESLEY: He was pleased to see that Mr. Livesley, who was an authority on machine computation, entirely agreed with his view that it was easier to set up the stiffness matrix than the flexibility matrix.

Regarding the suggestion that it might be worthwhile to convert the asymmetric dynamical matrix into a symmetrical form because that often made it easier to extract the latent roots, the consensus of opinion seemed to be that for very large matrices this was probably the case, since it reduced the storage room required. The advantages of such a conversion for matrices of moderate size were less obvious and, for the A.C.E. at the N.P.L. was, he understood, seldom resorted to.

Everybody would be interested in Mr. Livesley's point that, given adequate storage facilities, there was no reason why the machine should not be programmed to compute the coefficients and work out the frequencies and modes entirely on its own, having been fed only with the basic data of size and disposition of the structural elements.

DR. BATEMAN: He was very interested to see from Dr. Bateman's remarks that they might now look forward to the time when even the programming of the machine became largely an automatic operation.

MR. SHEVLOFF: In his opening remark Mr. Shevloff said he would like to challenge him on one point. Mr. Shevloff did not question the correctness of any statement in the paper, but merely suggested that the procedure outlined might at one particular point be shortened, with a consequent advantage in cost of machine time. It was clear that Mr. Shevloff had forgotten what was said in the introduction to the paper, where he emphasised that "my main object is to expound the general principles of the new approach rather than try to describe or compare the various detailed procedures and so on." That being his object, he forebore from confusing the general argument by pointing out various short cuts. The particular one pointed out by Mr. Shevloff depended on using equations (45) and (46) to express the R.H.S. of (42) in terms of the relative displacements appearing on the L.H.S. It was a useful short cut and standard practice at the N.P.L.

New Zealand Division

Fifth Annual Report: For Year Ended 31st December 1953

DURING the year the Branch activities of this Division have continued at a satisfactory level and membership has again shown a steady increase.

There have been three aviation functions of major interest during the year, namely, the commemoration of the 50th Anniversary of the Wright Brothers' First Flight, the Canterbury Centennial Air Race and the Walsh Memorial Air Pageant. To commemorate the Wright Brothers' First Flight a special function was held in Wellington for members and their wives. The Divisional Council is indebted to the Wellington Branch, who were the official hosts and organisers, and to the Shell Company and the Vacuum Oil Company who provided the films. On the following evening these special films were shown to the general public under the auspices of the Wellington Branch. The N.Z. Division itself was not directly connected with the Air Race, but many of its members were closely associated with the internal administration. An outstanding event of the Auckland Branch was the sponsoring of the Walsh Memorial Air Pageant. This was a very successful project in every way and was carried through with the wholehearted co-operation of the R.N.Z.A.F. and many prominent Auckland citizens.

The Division was gratified to receive an allocation of seats for the Address given by His Royal Highness The Duke of Edinburgh to representatives of scientific societies in Wellington. Members travelled from all centres for this occasion and were rewarded by a very interesting and stimulating Address.

A project involving the circulation of Aeronautical films of educational interest to secondary schools throughout the Dominion has been started. Results so far indicate that the scheme is being well received. It is intended to continue this work, although there are major difficulties with the time involved in circulating the films from school to school and with the very limited supply of suitable films. In addition the Division is sponsoring aircraft design study groups, who will compete for prizes offered by the Division.

A scheme for the circulation of pre-prints of Papers being presented before the Society in London has been initiated and several contributions to the discussion on these Papers have appeared in the Society's JOURNAL.

Branches this year have had to face the problem of the difficulty of obtaining suitable lecturers for their range of meetings. The necessary enthusiasm of Branch members can only be maintained at the required level if interesting and informative lectures are provided at regular intervals. A number of overseas aeronautical people, who could provide excellent material, visit us during the course of the year but their visits are either very brief and not known to the Society, or their itinerary is so closely scheduled that the inclusion of a lecture to any one of the Branches is impracticable. Both the Divisional Council and the Executives of the Branches are investigating ways and means of overcoming this problem.

Where meetings have either to be augmented or based on a screening, the N.Z. Division is particularly grateful to the Shell Company, the Vacuum Oil Company and Caltex for their co-operation and the use of their excellent film libraries.

During the year the activities of our Treasurer have been concentrated on a special effort to collect some of the rather large percentage of outstanding subscriptions. He has been particularly successful and has also devised an improved system of advising members when subscriptions are due.

The Divisional Council for 1953 was elected by postal ballot from all members known to be resident in New Zealand and the following were elected to serve:—

President
C. W. Labette, *Associate Fellow*

Vice-President
Air Commodore G. Carter, *Associate Fellow*

Hon. Secretary
T. T. N. Coleridge, B.E., *Associate Fellow*
Hon. Treasurer and Assistant Secretary
O. A. Kemp, A.R.A.N.Z., A.C.A.I.

Council Members
Sir Arthur Nevill, K.B.E., C.B., M.Sc., *Associate Fellow*
E. A. Gibson, O.B.E., *Associate Fellow*
C. G. Andrews, D.F.C., *Associate Fellow*
F. F. J. Butler, M.B.E., *Associate Fellow*
B. Cornthwaite, *Associate*
G. N. Roberts, C.B.E., *Associate.*
H. F. T. Adams, M.A., *Associate Fellow*
S./Ldr. G. S. Evatt, *Associate*
J. S. Brown, *Associate*

AUCKLAND BRANCH

Chairman: D. J. Patterson, M.B.E., *Associate*
Hon. Secretary: A. Jardine, *Associate Fellow*
Committee: L. G. Brister, *Associate*; E. Paton; A. Vause, *Associate*; A. Jardine, *Associate Fellow*

In addition to the Walsh Memorial Air Pageant the Branch held their Annual Aviation Interests Cocktail Party. Monthly meetings were also held, at which the average attendance was 37. In connection with the Memorial for the Walsh Brothers the Branch ran an essay competition among school children.

The total membership of the Branch is 141. In spite of 7 having left the district during the year the membership has risen from 130 last year.

PALMERSTON NORTH BRANCH

Chairman: M. R. Roper, *Associate Fellow*
Vice-Chairman: S/Ldr. G. S. Evatt, *Associate*
Hon. Secretary: J. S. Brown, *Associate*
Hon. Treasurer: Miss S. T. Murphy
Committee: J. R. Burford; A. W. Dingle, *Associate Fellow*; R. H. Nokes; R. Westcott

Over the year the average attendance at meetings was 39 and while the total Branch membership has risen from 78 to 130 a large transfer of staff from National Airways Corporation, Palmerston North, to Christchurch has meant a large drop in the members now in the district.

WELLINGTON BRANCH

Chairman: B. Cornthwaite, *Associate*
Hon. Secretary: M. R. Griffith
Hon. Treasurer: M. L. Clarke
Committee: C. G. Andrews, D.F.C., *Associate Fellow*; A. M. Harvie, A.F.C.; Flt. Lt. R. B. Bolt, D.F.C., *Associate*; F. F. J. Butler, M.B.E., *Associate Fellow*; C. W. Labette, *Associate Fellow*

Excluding the two public film evenings the average attendance at meetings was 37. The total membership has increased during the year from 120 to 159.

CHRISTCHURCH BRANCH

Chairman: H. Adams, *Associate Fellow*
Hon. Secretary: A. Brazier, *Associate Fellow*
Hon. Treasurer: F. Brooks, *Associate*
Committee: N. Mason; M. Seay; T. Moseley

The activities of the Branch have been severely restricted but an improvement in the coming year is anticipated because of a movement from Palmerston North. In spite of difficulties three meetings were held, two as combined meetings with the Automotive Institution. The total membership of the Branch at present is 31.

Membership of the Division

	As at Jan. 1953	*As at Jan.* 1954
Fellows	—	—
Associate Fellows	28	27
Associates	55	64
Graduates	7	7
Students	1	1
Companions	3	3
	94	102

Papers and Lectures

The papers and lectures given throughout the Branches during the year are listed below. Some were given at more than one Branch.

Rockets and Space Travel by G. W. Horne.
Some Direction Finding Methods for Aircraft by Dr. H. A. Whale.
Some Experiences with Oblique Photography in N.Z. by Leo White.
Discussion on Helicopters, their Uses, Limitations and Development.
Nuclear Energy by Sir Arthur Nevill.
Space by L. R. H. Beaumont.
Some Unusual Aspects of Turbulence by Captain F. Bethwaite.
Aeronautical Communications by D. F. Jenkins.
Some Engineering Conceptions of Coral Structures by A. M. Prichard.
Maintenance Problems of Vampire Aircraft by Flt. Lt. G. Cotterill.
Post War Trends and Developments in British Aircraft by M. R. Roper.
Some Impressions of Aviation Overseas by Captain J. J. Busch.
Helicopters by T. R. Pike.
Refuelling Problems of Comet Air Liners by L. Clarke.
The Canterbury Centennial Air Race—Route and Planning by Flt. Lt. Tosland.
Aviation Gas Turbines by A. C. Ellworthy.

Frequently films were shown at Branch meetings, either in conjunction with papers or as a separate function.

Divisional Annual General Meeting

The Statement of Accounts and the Annual Report for 1953 were adopted at the Annual General Meeting of the Division held on 8th March 1954. At that meeting the result of the postal ballot for the election of the New Divisional Council was announced, the following candidates being elected:—

H. F. T. Adams, *Associate Fellow* (Christchurch); C. G. Andrews, *Associate Fellow* (Wellington); Air Commodore G. Carter, *Associate Fellow* (Wellington); T. T. N. Coleridge, *Associate Fellow* (Wellington); B. Cornthwaite, *Associate* (Wellington); E. A. Gibson, *Associate Fellow* (Wellington); C. W. Labette, *Associate Fellow* (Wellington); Sir Arthur Nevill, *Associate Fellow* (Wellington); Wing Commander Woodward (Wellington); D. A. Patterson (Auckland); G. N. Roberts, *Associate* (Auckland); A. W. Dingle, *Associate Fellow* (Palmerston North).

STATEMENT OF RECEIPTS AND PAYMENTS FOR YEAR ENDED 31st DECEMBER 1953

Receipts	£	s.	d.
Funds on hand at 1/1/1954:—			
National Bank of New Zealand	371	12	10
Petty Cash	3	2	4
Received during year:—			
Subscriptions and Entrance Fees	376	10	9
Unexpended balance			
Aviation Industry Dinner	3	0	
	£751	8	11

Payments	£	s.	d.
Remittance to Royal Aeronautical Society, London, on account of 1952 subscriptions	96	10	2
Branch subscriptions for Division Members:—			
Palmerston North	1	0	0
Wellington	7	15	0
Canterbury	1	15	0
Grant to Palmerston North Branch for Film Evening Expenses	5	10	0
Grant to Palmerston North Municipal Library	15	0	0
Postage	13	10	7
Stationery	10	7	10
General Expenses	3	16	6
Bank charges and exchange		19	6
Funds on hand at 31/12/1953:—			
National Bank of New Zealand	594	4	3
Petty Cash	1	0	1
	£751	8	11

I have audited the accounts of the Royal Aeronautical Society, New Zealand Division, and the above is a true and correct statement of receipts and payments for the year ended 31st December 1953.

B. W. HOULT, B.Com.A.R.A.N.Z., *Hon. Auditor.*
O. A. KEMP, A.R.A.N.Z., A.C.A.I., *Hon. Treasurer.*

TECHNICAL NOTES

Contributions to this Section of the JOURNAL *will be eligible for Journal Premium Awards and will normally be published within two months of being received.*

Stability Criteria, with Special Reference to the Sextic Equation

PROFESSOR W. J. DUNCAN, C.B.E., D.Sc., F.R.S., F.R.Ae.S.
(Department of Aeronautics and Fluid Mechanics in the University of Glasgow)

STABILITY criteria for systems governed by linear differential equations with constant coefficients are discussed in a general way. Detailed treatment of the sextic equation includes a simple and expeditious scheme for computing the test functions.

1. INTRODUCTION

The aim of this Note is to discuss, in a general manner but very briefly, the stability criteria for systems whose differential equations are linear with constant coefficients and to demonstrate the incorrectness of some assumptions made about these criteria. As an example, the stability of systems whose determinantal or characteristic equation is of the sixth degree is considered in detail and a convenient scheme for computing the complete set of test functions in sequence is explained. Such sextic equations are of considerable importance in aeronautics since they arise, for example, in relation to all varieties of ternary flutter and in investigations of the stability of aircraft with a free control.

2. STABILITY CRITERIA WHEN THE CHARACTERISTIC EQUATION IS OF THE n^{th} DEGREE

When the differential equations governing the motion of a system are linear with constant coefficients they are solved by assuming that each dependent variable is proportional to $\exp(\lambda t)$, where t is the time and λ is a constant to be determined. The condition of compatibility of the equations which result from this assumption is the vanishing of a certain determinant and when expanded this yields an equation for λ of the n^{th} degree. Now the stability of the system depends entirely on the nature of the roots of this characteristic equation and the complete conditions for stability are that all the real roots shall be negative while the real parts of all the complex roots shall also be negative; in other words, all the radical points must lie on the negative side of the axis OY in the Argand diagram. Since an equation of the n^{th} degree has n roots, the number of independent conditions for complete stability is n. These rules hold even when the characteristic equation has repeated roots.

Before going farther it will be helpful to consider the distinction between *necessary* conditions for stability and *sufficient* conditions for stability. A set of conditions N is necessary when the violation of one or more of them certainly implies instability. On the other hand, a set of conditions S is *sufficient* when the satisfaction of them certainly implies stability. However, sufficient conditions may be too severe while necessary conditions may be insufficient or redundant. The ideal set of conditions NS is *necessary* and *sufficient* for stability. This means that the system is certainly stable when the conditions are all satisfied and certainly unstable when the conditions are not all satisfied. The conditions already stated regarding the real roots and the real parts of the complex roots are necessary and sufficient; the number of them is equal to the degree of the characteristic equation.

In order to apply the necessary and sufficient conditions for stability in the fundamental form already explained, it is necessary to solve the characteristic equation completely, *i.e.* to obtain the values of all its roots. However, Routh[1,2] showed that the set of necessary and sufficient conditions for stability could be put in the form that n *test functions* must be positive. These test functions $R_1, R_2 \ldots R_n$ were given by Routh as functions of the *coefficients* of the characteristic equation, which is supposed to be written with the coefficient of the highest power of λ positive. Moreover, the number of "unstable" roots (roots with their real parts positive) is equal to the number of changes of sign in the sequence of Routh's test functions with p_n as first term. Thus a complete test for stability can be made without obtaining the roots of the characteristic equation. It was shown by Hurwitz[3] and independently by Frazer[4] that a set of necessary and sufficient criteria (equivalent to Routh's) can be put in the form that the set of test determinants $T_1, T_2 \ldots T_n$ shall all be positive. Let the characteristic equation be

$$f(\lambda) \equiv p_n\lambda^n + p_{n-1}\lambda^{n-1} + \ldots + p_1\lambda + p_0 = 0, \quad . \quad (2.1)$$

where the equation is written with p_n positive.

Then
$$T_1 = p_{n-1} \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

$$T_2 = \begin{vmatrix} p_{n-1} & p_n \\ p_{n-3} & p_{n-2} \end{vmatrix} \quad . \quad . \quad . \quad (2.3)$$

$$T_3 = \begin{vmatrix} p_{n-1} & p_n & 0 \\ p_{n-3} & p_{n-2} & p_{n-1} \\ p_{n-5} & p_{n-4} & p_{n-3} \end{vmatrix} \quad . \quad . \quad (2.4)$$

and so on. The rule in constructing T_r (which is a determinant of order r) is: begin with p_{n-1} at the top left hand corner and write $p_{n-3} \ldots p_{n-2r+1}$ below it to form the first column. Then, beginning at the left hand

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end of any row, increase the suffix of the coefficient by one in passing from any element to its right hand neighbour. A zero must be entered when the suffix is negative or greater than n. It appears on writing out T_{n-1} and T_n that

$$T_n = p_0 T_{n-1}. \quad . \quad . \quad . \quad (2.5)$$

Hence the complete set of necessary and sufficient criteria for stability is that $T_1, T_2 \ldots T_{n-1}$ and p_0 shall be positive, where it is understood that the equation is written with p_n positive. It can be shown that

$$R_1 = T_1 \quad . \quad . \quad . \quad . \quad (2.6)$$

while

$$R_s = \frac{T_s}{T_{s-1}} \quad . \quad . \quad . \quad . \quad (2.7)$$

By use of these relations and Routh's rule the number of "unstable" roots can be ascertained. It can be shown[5] that T_r is homogeneous of degree r in the coefficients p and that all its terms have the same *weight* $\frac{1}{2}r(2n-r-1)$, where the weight of a term is the sum of the suffices of its factors. It can also be shown[5] that an equivalent set of criteria is obtained when p_{n-r} is substituted for p_r throughout the formulae*.

Routh[2] showed that the necessary and sufficient conditions for stability could be put in the form:

The roots of the equations

$$P(y) \equiv p_n y^n - p_{n-2} y^{n-2} + p_{n-4} y^{n-4} - \ldots = 0 \quad . \quad (2.8)$$

$$Q(y) \equiv p_{n-1} y^{n-1} - p_{n-3} y^{n-3} + \ldots = 0 \quad . \quad . \quad (2.9)$$

must all be real and the roots of each must separate or lie between those of the other.

The functions P and Q are obtained by making the substitution $\lambda = \pm iy$ in (2.1) and then separating the odd and even powers of y. As pointed out in section 3, a critical condition can be reached if (2.8) and (2.9) are satisfied simultaneously so that $P(y)$ and $Q(y)$ have a common root.

A *necessary* but *insufficient* condition for stability is that all the coefficients of (2.1) shall have the same sign, *i.e.* be positive when the equation is written with p_n positive (*see*, for example, Chap. 4 of Reference 5).

3. CRITICAL CRITERIA FOR STABILITY

Suppose that the system under investigation is completely stable in a standard state S and that one of its parameters is varied until a critical state C is reached, such that any further change of the parameter would result in instability. This can happen in two ways:

(*a*) In state C a real root of (2.1) which was negative attains the critical value zero. This is indicated by the vanishing of the constant term p_0 in the characteristic equation. In state C we have *neutral static stability* and on the unstable side of C there is a *real divergent motion.*

(*b*) In state C the real part μ of the complex pair of roots $\mu \pm i\omega$ attains the critical value zero. In state C the system can perform a simple harmonic oscillation when free and on the unstable side of C there is a divergent oscillation (oscillation of growing amplitude). Now in the critical state (2.1) is satisfied by the equal and opposite pair of roots $\pm i\omega$ and equations (2.8) and (2.9) are simultaneously satisfied by $y = \omega$. Consequently the eliminant of (2.8) and (2.9) vanishes; when this is obtained by Sylvester's dialytic method the critical condition is found to be[4]

$$T_{n-1} = 0. \quad . \quad . \quad . \quad . \quad (3.1)$$

The same critical condition can be written in many equivalent ways[4], but they all express the vanishing of an eliminant of equations (2.8) and (2.9). These equations are satisfied simultaneously whenever (2.1) is satisfied by equal and opposite roots $\lambda = \pm\beta$. Consequently T_{n-1} (or any other form of the eliminant) must contain as a factor Π, the continued product of the sums of the roots of (2.1) taken in pairs. It can be shown[5] that

$$T_{n-1} = (-1)^{n(n-1)/2} p_n^{n-1} \Pi; \quad . \quad . \quad (3.2)$$

the index of -1 is the number of factors in Π.

The critical condition (3.1) is of outstanding importance in relation to flutter since it is the equation which determines the critical speeds for flutter.

4. THE SEXTIC EQUATION

The equation under consideration is

$$f(\lambda) \equiv p_6\lambda^6 + p_5\lambda^5 + p_4\lambda^4 + p_3\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.1)$$

and we suppose that this is written with p_6 positive. In accordance with Section 2 *the complete set of necessary and sufficient conditions for stability is*

$$T_1 = p_5 > 0 \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

$$T_2 = \begin{vmatrix} p_5 & p_6 \\ p_3 & p_4 \end{vmatrix} > 0 \quad . \quad . \quad . \quad . \quad (4.3)$$

$$T_3 = \begin{vmatrix} p_5 & p_6 & 0 \\ p_3 & p_4 & p_5 \\ p_1 & p_2 & p_3 \end{vmatrix} > 0 \quad . \quad . \quad . \quad (4.4)$$

$$T_4 = \begin{vmatrix} p_5 & p_6 & 0 & 0 \\ p_3 & p_4 & p_5 & p_6 \\ p_1 & p_2 & p_3 & p_4 \\ 0 & p_0 & p_1 & p_2 \end{vmatrix} > 0 \quad . \quad . \quad (4.5)$$

$$T_5 = \begin{vmatrix} p_5 & p_6 & 0 & 0 & 0 \\ p_3 & p_4 & p_5 & p_6 & 0 \\ p_1 & p_2 & p_3 & p_4 & p_5 \\ 0 & p_0 & p_1 & p_2 & p_3 \\ 0 & 0 & 0 & p_0 & p_1 \end{vmatrix} > 0 \quad (4.6)$$

$$p_0 > 0. \quad . \quad . \quad . \quad . \quad (4.7)$$

The direct computation of T_4 and T_5 is rather laborious but the work can be put in a convenient form by using the following relations which are readily obtained by partial expansion of the determinants:

$$T_3 = p_3 T_2 - p_5 D_1 \quad . \quad . \quad . \quad . \quad . \quad (4.8)$$

where $$D_1 = p_2 p_5 - p_1 p_6 \quad . \quad . \quad . \quad . \quad . \quad (4.9)$$

$$T_4 = p_2 T_3 + D_2 T_2 + p_1 p_6 D_1 \quad . \quad . \quad . \quad (4.10)$$

*In numerical work the equation is usually written with $p_n = 1$. However, it is most undesirable to take p_n as unity in the general theory since important features of the structure of the test functions are then completely masked.

where $D_2 = p_0 p_5 - p_1 p_4$ . . . . . (4.11)

$$T_5 = p_1 T_4 - p_0 p_3 T_3 + p_0 p_1 p_5 T_2 - p_0^2 p_5^3 \quad (4.12)$$

$$T_6 = p_0 T_5. \quad (4.13)$$

The scheme of computation is therefore as follows: calculate in succession, making use of the above relations,

$$T_2,\ D_1,\ T_3,\ D_2,\ T_4 \text{ and } T_5.$$

Except perhaps in the case where the investigation is confined to the search for a critical condition it is advantageous to compute the complete set of test functions. Often the existence of instability is revealed by the negative sign of one of the *early* test functions. In order to find the number of unstable roots we count the number of changes of sign in the sequence $p_6 p_5 R_2 R_3 R_4 R_5 p_0$ where the signs only need be found from the relations (2.7).

Of the many alternative expressions[4] for T_5 the following is worthy of mention on account of its simplicity

$$T_5 = -p_5^3 (p_6 y_1^3 + p_4 y_1^2 + p_2 y_1 + p_0)(p_6 y_2^3 + p_4 y_2^2 + p_2 y_2 + p_0) \quad (4.14)$$

where y_1 and y_2 are the roots of the quadratic equation

$$p_5 y^2 + p_3 y + p_1 = 0. \quad (4.15)$$

It is clear that T_5 cannot be positive when the roots of the last equation are complex (compare Routh's theorem given at the end of Section 2) and p_5 is positive, for the product of conjugate complexes is certainly positive. When all the coefficients p are positive T_5 will be negative unless the roots of (4.15) are *real and negative*[4]. We see that, with p_5 positive, T_5 is certainly negative and the system unstable when

$$p_1^2 - 4 p_1 p_5 < 0. \quad (4.16)$$

It has been asserted by various persons that the system must be stable when all the coefficients of the sextic and T_5 are positive*. This is untrue and it may be well to demonstrate the untruth explicitly.

The formula (3.2) becomes here

$$T_5 = -p_6^5 \Pi. \quad (4.17)$$

Now let us suppose that all the roots are complex and given by $\mu_1 \pm i\omega_1$, $\mu_2 \pm i\omega_2$, $\mu_3 \pm i\omega_3$. Then it easily follows that Π, the continued product of the sums of the roots taken in pairs, is given by

$$\Pi = 8\mu_1\mu_2\mu_3 K \quad (4.18)$$

where K is necessarily positive, its factors being the sums of squares of real numbers. Accordingly (4.17) becomes

$$\frac{T_5}{p_6^5} = -8\mu_1\mu_2\mu_3 K. \quad (4.19)$$

Now p_6 is taken positive and a negative value of μ corresponds to "stable" roots. Hence we see that T_5 is

Positive with *no* unstable roots or with *two* unstable pairs

Negative with *one* or *three* unstable pairs.

Further, as will be shown, all the coefficients of the sextic will be positive when we have one stable pair of roots and two unstable pairs, provided the latter are not too highly unstable. Hence we can have T_5 and all the coefficients positive when there are two pairs of unstable complex roots. In such circumstances the sextic can be resolved into real quadratic factors as follows

$$f(\lambda) = p_6 (\lambda^2 - e\lambda + a)(\lambda^2 - f\lambda + b)(\lambda^2 + k\lambda + j) \quad (4.20)$$

where p_6, a, b, e, f, k and j are all positive. For the moment suppose that e and f are zero. Then (4.20) yields on expansion a *complete* sextic whose coefficients are *all* positive; when e and f are restored the coefficients remain positive provided that e and f are not too large.

We shall illustrate this point and the calculation of the test functions by taking

$$f(\lambda) = (\lambda^2 - \lambda + 4)(\lambda^2 - 2\lambda + 10)(\lambda^2 + 5\lambda + 10)$$
$$= \lambda^6 + 2\lambda^5 + 11\lambda^4 + 32\lambda^3 + 110\lambda^2 + 20\lambda + 400 \quad (4.21)$$

whose coefficients are thus all positive. We follow the scheme of computation already explained and obtain

$$T_2 = -10$$
$$D_1 = +200$$
$$T_3 = -720$$
$$D_2 = +580$$
$$T_4 = -8{\cdot}1 \times 10^4$$
$$T_5 = +6{\cdot}156 \times 10^6.$$

Thus T_5 and all the coefficients are positive but the equation has two pairs of unstable complex roots. The roots of the equation are explicitly $-2{\cdot}5 \pm 1{\cdot}9365i$, $+0{\cdot}5 \pm 1{\cdot}9365i$, $+1{\cdot}0 \pm 3{\cdot}0i$. We find that the signs of the sequence $p_6 p_5 R_2 R_3 R_4 R_5 p_0$ are $+\ +\ -\ +\ +\ -\ +$. Hence there are four changes of sign and four unstable roots, which is correct.

It is worthy of remark that the existence of instability in this example is shown at the beginning of the numerical investigation by the negative sign of T_2.

*It happens to be true for the quartic that the system must be stable when all the coefficients and T_3 are positive.

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The Manufacture of Glass Cloth/Resin Blades for an Axial Flow Compressor

by

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GLASS CLOTH/POLYESTER resin laminates similar to those described by Irving and Saunders (JOURNAL, February 1954) have been used at the University Engineering Laboratory, Cambridge, in the manufacture of blades for an axial flow compressor.

In the first experiments, an existing aluminium stator blade was used as a pattern, and a mould was made by pouring molten type metal round this pattern, held in a steel sided moulding box. The leading and trailing edges of the blade touched the sides of the moulding box and formed the parting line of the mould. Rubber seals were provided at the join of the steel sides of the box, and the mould surfaces were polished with wax. Five strips of glass cloth were placed in the mould and the resin slowly fed in from two points at the base of the mould, which was left open at the top to enable air bubbles to escape. The blades were cast at room temperature, taking two or three hours in setting, after which they were heated in the mould at 80°C. for half an hour.

The resin has also been cast around steel or aluminium root fixings; alternatively the root shape may be machined from the block of plastic.

Finally the blades were painted to give the high surface finish required.

These plastic stator blades were tested in an axial flow compressor at the Engineering Laboratory, Cambridge. The rotational speed of the compressor during the test was 6,000 r.p.m. (tip diameter 14 in.), and air velocities up to 200 ft./sec. were observed. The aerodynamic performance with plastic stators was identical with the performance observed when aluminium stators were used and inspection of the blades after several hours of running under both stalled and unstalled operating conditions of the compressor revealed no flaws in manufacture.

Following the success in the manufacture of the stators, a pattern for a rotor blade was made, and glass cloth/resin rotor blades were produced in the same way, the fir tree root fixing being cast in plastic. The rotor blades were tested in the compressor at 4,500 r.p.m.

The manufacture of these blades was completed in the Instrument Shop of the Cambridge University Engineering Laboratory Workshops.

Received 7th April 1954

A Proposal on the Electric Tank Method for Study of a Symmetrical Aerofoil with a Rear Suction Slot and a Retractable Flap

by

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AN EXPERIMENTAL study of fluid flow past a symmetrical aerofoil with a rear suction slot and a retractable flap was described by D. M. Heughan in the October 1953 JOURNAL. The present note describes the electric tank method for studying the same problem.

The experimental apparatus is shown in Fig. 1. *A* and *B* are electric poles along two of the edges of the tank which is filled with water. The poles *A* and *B* together with the central pole *P* are connected to an electric supply via a resistance *R* in such a way that the potential of *P* is intermediate between that of *A* and *B*. The aerofoil *F* is represented by a wooden block coated with paraffin wax and the pole *P* is located in such a way as to represent the sink in the suction slot. A voltmeter *V* records the potential at a point in the tank.

When the fluid flow has a vortex or circulation in it, the positions of the poles *A*, *B*, must be so chosen that the equipotential lines coincide with streamlines as shown in Fig. 2(*a*). If there is a source or sink in the flow, the equipotential lines are perpendicular to the streamlines (Fig. 2(*b*)). In the case of the aerofoil with

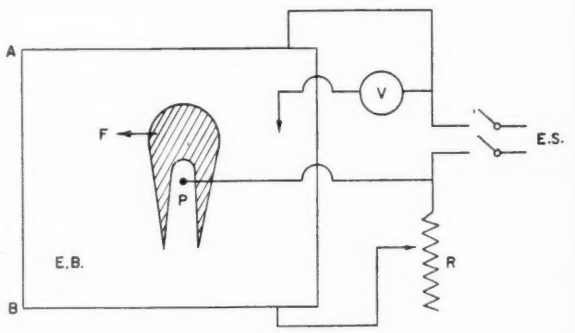

FIGURE 1.

Received April 1954.

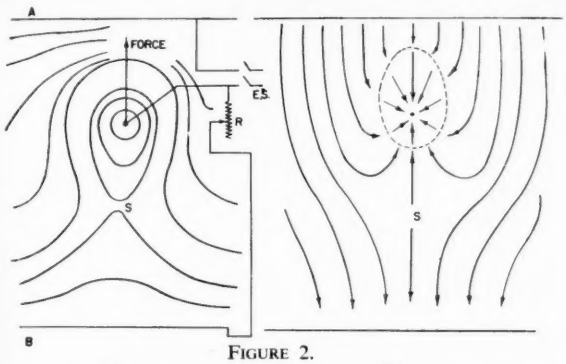

FIGURE 2.

(*a*) (*b*)

(*a*) Streamline parallel to equipotential line (Magnus effect) by electric bath method.

(*b*) Streamline perpendicular to equipotential line.

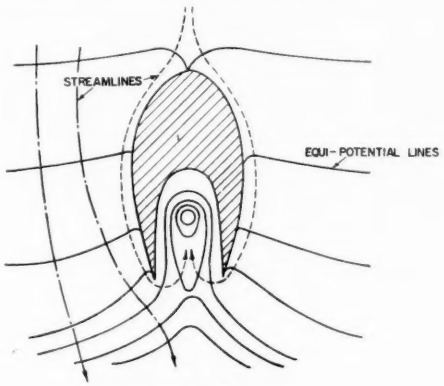

FIGURE 3.

the suction slot, the streamlines around the body are affected by the position of the pole *P* which represents the sink and this position must be carefully selected in the deeply entered part of the aerofoil.

As in this case the position of the poles *A* and *B* have been selected so that the streamlines are perpendicular to the equipotential lines, it is not possible to add circulation to the flow using this method. The method can therefore only be applied to the case of the non-lifting or symmetrical aerofoil.

Figure 3 illustrates the principle of the method and Figs. 4 and 5 are examples.

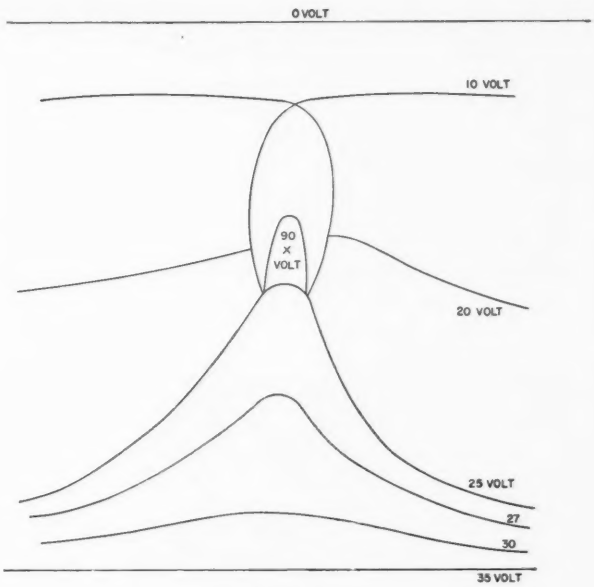

FIGURE 4.

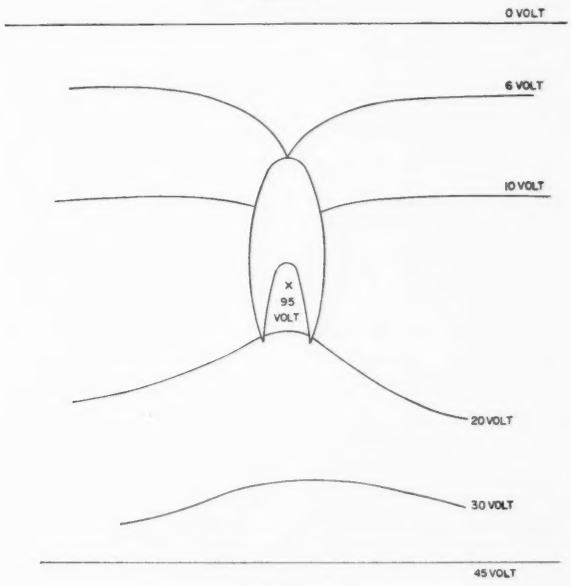

FIGURE 5.

A Photoelastic-Fatigue Programme of Experimental Research in Connection with Bolted Joints

By

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AN OUTLINE is given of a combined photoelastic and fatigue programme of work on single-pin double-strap joints; the stress concentration factors obtained from photoelastic tests are to be correlated with the results from a parallel series of metal fatigue tests on geometrically similar specimens. The form and dimensions for a photoelastic specimen suitable for this work are given. Further results will be published as the work proceeds.

Received April 1954

1. INTRODUCTION

This Note outlines a programme of work recently initiated by the Royal Aeronautical Society, and now being done under Ministry of Supply contract, in connection with the fatigue strength of bolted joints.

The bolted joint was selected for such an investigation for two main reasons:—

(*i*) It is representative of certain parts of the aircraft structure which are especially liable to failure from fatigue.

(*ii*) A bolted joint is a structural element having many regions of stress concentration. It is well known that points of static stress concentration form nuclei from which fatigue cracks will start; it was considered therefore that a systematic approach to this problem along such lines as those about to be described might yield information which would be of use in detail design.

The programme will consist of two parts:—

(*a*) A photoelastic investigation with the object of obtaining quantitative information on stress concentration for various kinds of joints.

(*b*) A parallel series of fatigue tests on metal specimens.

The results of the photoelastic and fatigue tests will be correlated; so that this can be done, the fatigue tests will be carried out with metal specimens having the same geometrical form as the photoelastic ones.

2. OUTLINE OF PROGRAMME

The complete programme will cover all the essential variables involved in the design of a joint, such as the following:—

Bolt and plate material
Bolt size and plate thickness
Bolt spacing and bolt pattern
Fit of bolt in hole
Clamping effect of bolt head and nut
Type and form of joint.

The major part of such an investigation will require the use of three-dimensional photoelastic technique, but a start is being made with some simple two-dimensional cases, as described in the next section.

3. OUTLINE OF INITIAL PROGRAMME

The initial series of tests will deal with stresses in a plate having a single hole and will cover the case of a simple single-pin double-strap joint (*see* Fig. 1).

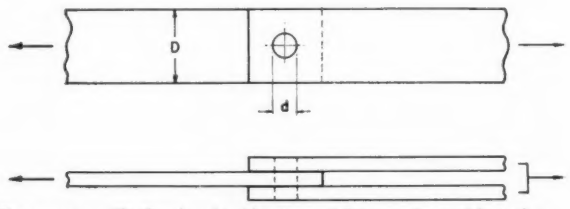


FIGURE 1. Single-pin double-strap joint to be subjected to repeated cycles of tensile load.

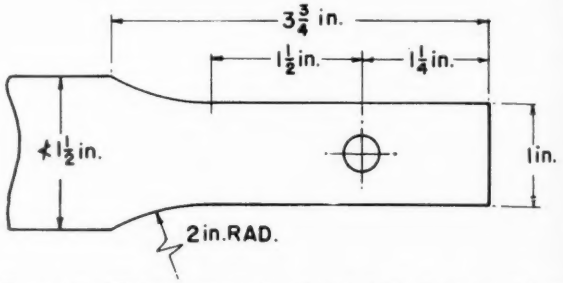


FIGURE 2. Minimum dimensions for photoelastic test specimen in which load is applied to the plate.

The photoelastic tests will explore the two-dimensional stress distribution under static load; the metal specimens in the fatigue tests will be subjected to a fluctuating range of tensile load. A range of load will be chosen that will have some realistic bearing on the aircraft joint design problem.

The following variables will be investigated:—

Ratio of diameter of hole to width of plate
Ratio of Young's moduli of pin and plate
Interference fit between pin and hole.

The photoelastic side of this programme has been started and some details of the initial tests are given in the next section.

4. INITIAL PHOTOELASTIC PROGRAMME

The first series of photoelastic tests being done is as follows:—

(*i*) *Preliminary tests*
To determine the form of specimen necessary to ensure uniform stress distribution in the test portion when there is no hole. The results of these tests are given in Section 5.

(*ii*) *Single-hole tests*

(*a*) *Unfilled hole: plate in tension*
These tests will be done for the following values of d/D:—

1/2, 3/8, 1/4, 1/8,

where d = diameter of hole
D = depth of plate.

(*b*) *Pinned hole: load (tensile) applied to plate only*
It is proposed to do these tests for the following conditions:—

d/D:	1/2, 3/8, 1/4, 1/8
E_1/E_2:	1, 3 and approximately 100*
Fit of pin:	(*i*) Push fit
	(*ii*) Two values of interference fit
where:	E_1 = Young's modulus for pin material
	E_2 = Young's modulus for plate material.

*These values represent the cases of (*i*) plate and pin of similar material; (*ii*) steel pin in light alloy plate; (*iii*) steel pin in photoelastic model.

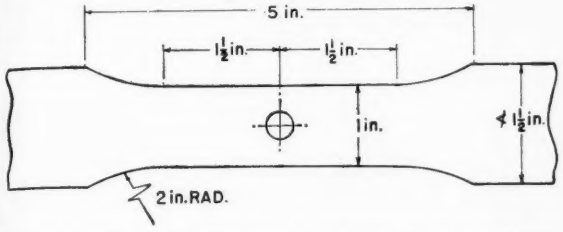


FIGURE 3. Minimum dimensions for photoelastic test specimen in which load is applied to the pin only.

(*c*) *Pinned hole: load (double shear) applied to pin only*

These tests will be done for the same values of d/D, E_1/E_2 and fit of pin in hole, as for the last case.

(*d*) *Pinned hole: loads applied to plate and pin*

The tests under this heading are for the purpose of checking whether the results from (*b*) and (*c*) can be superimposed without serious error, and at this stage it is not considered necessary to do here the full range of tests outlined for (*b*) and (*c*). It is intended to cover the following conditions:—

d/D: 3/8, 1/4

E_1/E_2: 1, 3

Fit of pin: (*i*) Push fit

(*ii*) One value of interference fit.

This part of the work forms an introduction to the case to be investigated later—of a multi-bolt joint—in which part of the load is transmitted by the bolts and part by the plate.

In the initial tests, equal loads will be applied to plate and pin.

5. THE PHOTOELASTIC TEST BAR

Tests have been carried out to determine a suitable shape of specimen to be used in both photoelastic and fatigue tests.

It was decided to use as model a uniform bar of rectangular section, the ratio of depth to thickness being 8, and the ratio of depth to diameter of hole varying from 8 to 2.

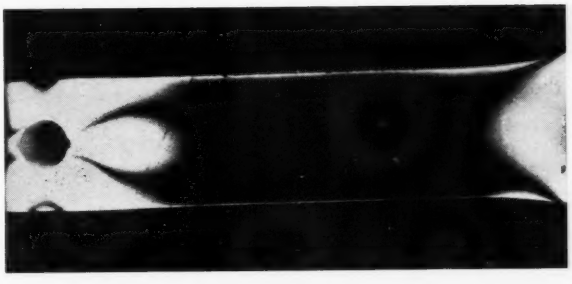
Two types of bar had to be considered: the first for cases where an overall tension was to be applied to the bar, the second for cases where the load in one direction was applied to a pin (or pins) passing through the hole (or holes).

The essential conditions to be achieved are:—

(*i*) The uniform portion of the bar must be sufficiently long for the stress distribution in the neighbourhood of the hole(s) to be unaffected by end conditions.

(*ii*) The bar must be designed so that in the fatigue tests failure will occur first in the neighbourhood of the hole(s).

(*a*)



(*b*)

FIGURE 4. Stress distribution in photoelastic bars subjected to a tensile load.

The minimum dimensions for these conditions to be fulfilled in the two cases are shown in Figs. 2 and 3.

When two or more holes are used, the distances of the fillets (or of the free end) from the centre of the nearest hole should be as shown in these figures.

The length of the uniform portion of the bar may be increased without affecting the stress distribution around the hole, and in the photoelastic tests this is being done in order to obtain a region in which the stress in the bar remains sensibly uniform. This will allow of direct measurement of the stress-concentration factor without having to determine the fringe-value of the material by a separate experiment.

These points are illustrated in the photographs reproduced in Fig. 4. Fig. 4(*a*) shows the stress distribution in a bar measuring 6 in. between loading holes, and Fig. 4(*b*) shows the stress distribution in a bar 8 in. between loading holes, with a hole drilled at 3 in. from one end.

The dimensions of the photoelastic model being used in this investigation are shown in Fig. 5.

It will be noted that the specimen needs to be increased in length only on one side of the hole.

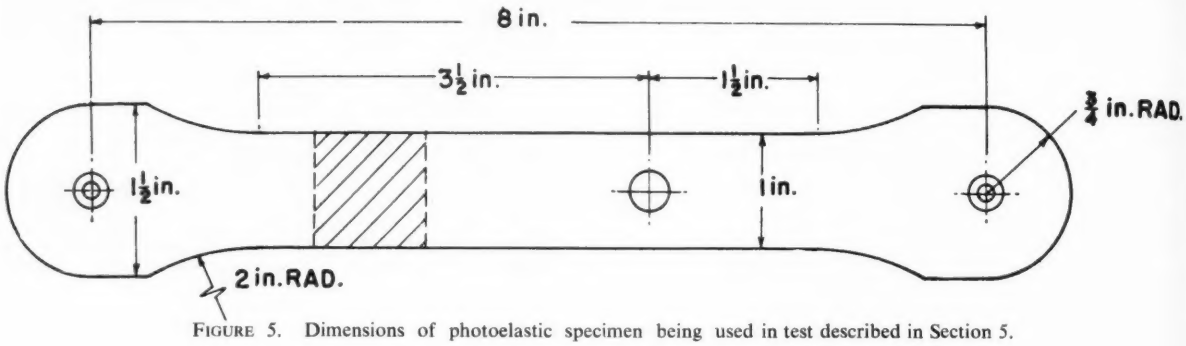


FIGURE 5. Dimensions of photoelastic specimen being used in test described in Section 5.

ACKNOWLEDGMENTS

The authors wish to thank the Ministry of Supply, the Royal Aeronautical Society and University College, London, for permission to publish the information contained in this Note, and for the help received from all concerned.

Stress Analysis of Circular Frames

by

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IN MY NOTE on "Stress Analysis of Circular Frames"[1] I was unable, due to lack of space, to discuss more thoroughly the reasons why the name of W. Ritz is omitted with reference to his method. The reader, interested in that subject, can find all the details in my recently published discussion[2] and paper[3].

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Reviews

ATOMIC ENERGY. *A Survey. Edited by J. Rotblat. Published for Atomic Scientists' Association by Taylor and Francis Ltd., London. 1954. 4s. 6d. paper; 6s. 6d. cloth.*

In January and February 1954 the Atomic Scientists' Association organised a series of seven public lectures at the University of London. The object of these lectures was to provide an up-to-date survey of the more important aspects of atomic energy, together with a sound assessment of the prospects of the future, by acknowledged authorities in their field, a survey intended for the general public.

These lectures are of primary importance to everybody and should be widely read.

In a preface to these lectures Professor Rotblat says of the subject of Atomic Energy, " It has created new fields of research and advancement and has posed to the public many fundamental issues concerning security, welfare and ethics. In order to tackle these problems sensibly it is necessary to have an informed public opinion, and the prerequisite for this is a fair knowledge of the facts concerning atomic energy."

The aims of the Atomic Scientists Association, formed in 1946 by scientists who had taken part in war-time Atomic Energy Projects, are to inform the public of the technical aspects and potentialities of Atomic Energy and, through its Journal, provide the means of discussing the social, political and international implications.

It is refreshing to find that some scientists have banded together, not only to discuss the work they do and to inform others, including the public as far as possible of that work (for this is the aim of most scientific and technical organisations), but to discuss, also, political and international aspects of their work (something which nine out of ten scientific and engineering associations avoid with conscious rectitude as not being one of their functions).

In the first lecture Sir John Cockcroft reviews Atomic Energy Research at Harwell. In six brief years Harwell has made great strides in the production of radioactive isotopes, now used throughout the scientific, medical and industrial worlds. There is an astonishing table of annual shipments of these isotopes to most parts of the world from Harwell. Here are physicists, scientists and engineers working on projects which, even at this early stage, are clearly destined to alter the lives of the human race. On the instrument side the Electronics Division produces many kinds of remarkable electronic instruments for measuring quantities which were quite beyond the range of measurements but a year or so ago. For example, a time interval of a thousand millionth of a second can be measured, or electric currents of as low as a hundred electrons a second (less than 10^{-16} of an ampere).

Professor Frisch, of the Cavendish Laboratory, Cambridge, discusses the principles of explosives, and comforts the layman with the statement, " We need not worry that the high temperature created in a hydrogen bomb may 'set fire' to the surrounding air, water or rock, and thus destroy all life on earth. That possibility has been carefully looked into and can be excluded with complete confidence."

Professor F. E. Simon, of the Clarendon Laboratory, Oxford, discusses possible applications in his lecture on Power from Atomic Energy.

" There is quite a lot of publicity given to the use of nuclear power in aircraft. . . . Doubtless, by lavishing funds and man power on it one will get a pile to take the air in, say the next ten years, but at the best, I think, this venture is going to end up as a white elephant on one's hands. The position may, however, look different at the end of the century."

I well remember this possibility being secretly discussed nearly ten years ago by a Committee of the Society, and I think some of us looked forward hopefully to an atomic engine weighing round about 15 tons, and developing horsepowers beyond the dreams of the most optimistic of aircraft and engine designers.

But that was ten years ago!

White elephants for aeroplanes, but not even pink ones for those drunk with enthusiasm for trips to the moon.

" There is a lot of speculation about the use of atomic powered rockets for space flight," says Simon. " The rocket will, however, have to carry a considerable weight of other materials which may be ejected to provide the necessary momentum, and this does away with the main advantage of nuclear fuel. Our space cadets will, therefore, have to rely on the 'chemical fuels' for their escapades."

But as a new source of energy for the production of electrical power Professor Simon is more optimistic, for he expects the cost to come down to the same level as that of coal-produced electricity. The technical problems awaiting the utilisation of atomic energy are many and difficult.

" One thing is certain: those who believe that atomic energy will usher in a new age of power for the asking, and provide the remedy for all those ills which their own inertia and lack of vision still prevent them from tackling, are in for a rude awakening."

Brave and necessary words, my masters! This lecture alone is worth the money, and the lecturer's remarks on waste, out-of-date methods of working and of thought, research, technological training and the like show an attitude of mind which is so sadly lacking in the so many, whether planners or planned.

" We have always had the coal wasters," says this not-so-simple Simon, for example, " who poison our atmosphere on the pretence that no Englishman can live without an open fire; now we have in addition the electricity wasters who act on the principle that if you are short of bread then you should eat cake."

It may be noted here that the development of nuclear power is already the largest single industry in the United States, and is getting pretty big in the United Kingdom.

Dr. Loutit's lecture on Radiation Hazards indicates how serious they may be, and how difficult it is to get rid of the radioactive products of nuclear reactors. His last words on the subject are, I fancy, a little depressing.

" The ocean deeps, which are relatively undisturbed, may well provide a suitable burial ground for concentrated sources in appropriate coffins, until technical improvements in propulsion allow us to discharge them into interplanetary space."

There comes some comfort from Dr. Pochin's lecture on the Medical Uses of Atomic Energy. " The isotopes are barely ten years old," he points out, " yet already we

have a variety of tests of the very highest value in research, and in a number of instances, of great convenience and some importance in routine clinical medicine."

What you gain on the isotopic swings you lose on the radiating roundabouts, as it were.

The sixth and seventh lectures by Professors Kathleen Lonsdale, F.R.S., and Sir George Thomson, F.R.S., are on the same subject, Atomic Energy and Moral Issues. It is an assessment of the importance attached to the subject by the organisers of the series, that two such distinguished speakers should be asked to give their views, on an aspect of scientific and technological development not usually publicly lectured upon by scientists or engineers.

" As citizens we ought to be discussing these questions vigorously," declared Professor Lonsdale. " One of the reasons why we do not discuss them is that we do not know the facts. The facts are kept secret for security reasons. I believe this secrecy is futile and harmful, and I believe that it is both futile and harmful *because* it is immoral. . . . That secrecy is harmful to science I believe there can be no doubt. . . .

" It is dangerously possible for scientists, as for other men, to anæsthetise themselves into supposing that to do as they are told to do or as they are paid to do by a democratically elected government or by a popularly-supported government whether elected or not, is in fact the right and moral thing to do. . . . Democracy is government by discussion. . . . You and I take part in that discussion as citizens . . . and it is our duty to do so and make our opinions known. It is equally our duty, since we are men and not machines, to consider all the implications of our work and of our national policy and to decide for ourselves the moral issues involved. Then we must act accordingly even if we act alone."

The gulf is a wide one between those who take up the attitude expressed by Professor Lonsdale and that expressed by Sir George Thomson. " I want to make what I have to say as practical as possible and avoid general issues," he begins but declares that he finds it difficult to come to a practical conclusion what to do. But when he says, " If the Russians were so foolish as to start bombing western cities, I believe we are justified in retaliating—but it ought to be true retaliation, by which I mean only doing enough to persuade them to stop, and making it quite clear we would stop when they did," he does not add to his reputation, for this is the eye for an eye and the tooth for a tooth argument of, one had hoped, a forgotten period in the history of Homo Sapiens.

With all these experts on electrons, neutrons and protons expressing so divergent views how can the poor human moron decide what to do? It may be that Mr. Gordon Dean, until recently Chairman of the United States Atomic Energy Commission, was right when he said, " It does no good to reach the point where we would be able to wipe out an enemy twenty times over, if he reaches the point where he can wipe us out just once."

To the reviewer it is a source of great gratification that these lectures should have been suggested by Sir Roderic Hill, an aeronautical man of some eminence and Vice-Chancellor of the University of London. They are lectures which it will well repay aeronautical men of lesser eminence to study.—J. LAURENCE PRITCHARD.

ROYAL AIR FORCE 1939-1945. *Volume I—The Fight at Odds. Denis Richards. H.M.S.O. 430 pp. Photographs, maps and diagrams. 13s. 6d. net.*

This is the first of three volumes of the R.A.F. 1939-1945 War History written by Denis Richards and the late Hilary St. George Saunders.

This first volume is by Richards, who headed the official group of historians and technical experts writing confidential reports for the Air Ministry. It opens in 1934 with the beginning of rearmament and covers the war in France and Norway, the Middle East and the Atlantic up to the end of 1941.

To write a comparatively short War History of the R.A.F., with so many official documents from which to choose, may at first seem simple, but on second thoughts it becomes a formidable task to know just which, of so many interesting reports, to incorporate into the text. Denis Richards has achieved the right choice in no small way, thus making the history both interesting and easy to read. It is refreshing, too, to read the combat reports of the lesser known pilots rather than, as we so often see, the reports of the well known " Aces " of the Battle of Britain. I have, however, one small criticism to make; although we are forewarned that the author alone is responsible for the views expressed, I had the feeling throughout that he was a little harsh in his assessment of the Bombers' efforts in the early stages of the War and, more especially, of the R.A.F. efforts as a whole at Dunkirk. In speaking of the night attacks on precise objectives in Germany, such as aircraft factories, oil plants, marshalling yards, etc., he says:—

> " Doubtless it was somewhat optimistic to expect any great result from the fifty or so aircraft which carried out these tasks every night. Doubtless too, much of the bombing went astray."

Is this pure assumption really fair?

Of the advance of the Greek forces into Albania the author says that the British Commander kept his air power concentrated on the two main ports of Durazzo and Valona and then adds:—

> " On the whole, captured enemy soldiers showed a healthy respect for our raids, and one group, taken in January within twelve miles of Valona, confessed that the supply situation had become so bad that they received food only once in every three or four days. Doubtless this was due more to the Italian Military system than to our bombing, but the latter undoubtedly helped."

This seems a little disparaging.

However, this is perhaps just a small difference of opinion and does not prevent me from recommending the book as an excellent record of the history of the Royal Air Force in the early stages of the War and therefore a valuable addition to anybody's bookshelf.—W. R. HARTWRIGHT.

PROCEEDINGS OF SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS, VOL. XI, NO. 1. *Cambridge, Mass., 1953. 248 pp. Illustrated.*

Among the papers contained in this number of the Proceedings, that on " Effects of plastic flow and work-hardening in the experimental stress analysis of magnesium-alloy parts " should hold as much interest as any for aeronautical engineers. In view of the undoubted attractions of magnesium as a structural material, the account given in this paper of a careful experimental investigation of its stress-strain properties under various conditions of loading must be regarded as valuable new data for assessing its fatigue properties. Another paper of " fatigue " interest describes how existing types of fatigue-testing machines were successfully converted to enable bending-cum-torsion fatigue tests to be carried out,

a special feature of the resulting constant amplitude machine being the automatic stopping of the test at a pre-arranged percentage loss of stiffness of the test specimen.

Of interest also are two papers that describe ad hoc structural investigations, one into the cyclic stresses occurring in the blades and root assembly of a helicopter, and the other into the complicated stress field of a magnesium-alloy truck-wheel under various working conditions. The wheel analysis should have special interest in that it was specifically directed towards making the wheel lighter without making it weaker.

There are a number of papers describing new or improved instruments and techniques. Among new instruments may be mentioned two types of electrical resistance strain gauges, one to provide self-compensation against temperature changes and the other to accommodate the large strains met with in the plastic range. One of the new techniques described is the use of pulse instead of continuous excitation of strain gauges in dynamical multi-channel observations—a type of excitation that allows the output from several gauges to utilise the same amplifier. Another new technique concerns a method for choosing the best combination of diaphragm and strain gauge for pressure capsules intended for various operational conditions.

The above papers constitute about half the total number in the present volume. The other half in my view are, for one reason or another, a little disappointing and hardly up to the standard one has come to associate with the Society's Proceedings. Too many of these describe methods and techniques that have nothing but their ingenuity to recommend them. This is particularly true of some of the analogues described, one of which indeed involves such a complicated contraption that it can hardly have been intended as a serious alternative to existing methods. Some of the other papers are sub-standard because they describe devices the design of which one would imagine any competent laboratory assistant would take in his stride.—D. WILLIAMS.

THE AIRCRAFT ENGINEER'S HANDBOOK. *No. 4. Instruments. R. W. Sloley and W. H. Coulthard. 251 pp. and 178 illustrations. 6th edition. Pitman 1953. 30s. net.*

This book is written for the Aircraft Engineer who is intending to qualify for a Civil Aircraft Maintenance "X" Licence in Instruments.

Qualifications for an Aircraft Licence demand of necessity considerable practical experience, but the applicant for a Licence also has to sit for a written examination. In this examination he has to demonstrate his knowledge of construction, overhaul, repair, inspection and maintenance of non-electrical engine and aircraft instruments; of troubles which develop in operation and how to deal with them; of calibrating, adjusting and testing; and of the construction, operation and testing of sub-standard test equipment. In addition, he must show a knowledge of such elementary physics and arithmetical calculations as would be involved in these duties.

It is not easy to deal adequately with a subject which is so essentially a practical one, and the authors are therefore to be congratulated on producing a book which deals extremely well with those aspects covering construction, operation, testing, inspection and maintenance. Very wisely perhaps, no attempt is made to deal with the detailed repair of instruments. This is a side of the subject that is already dealt with in standard instrument manuals much more comprehensively than it could be here without unduly increasing the size of the book. An Appendix dealing with minor repairs and adjustments is included however.

Bearing in mind the purpose of the book, it might have been a good idea if the authors had included something dealing with some of the many troubles that commonly occur and how to remedy them, and with the special precautions that must be followed in the installation of instruments in an aircraft.

Although an "X" Licence can be obtained for instruments which function entirely mechanically, electrical instruments are used nowadays to such a great extent that no book of this kind would be complete unless it dealt with these too. The value of this book is therefore enhanced by containing matter on electrical and some electronic instruments.

It is assumed, quite rightly, that the reader has the necessary elementary knowledge of physics and mathematics as laid down by the Air Registration Board syllabus.

The publishers are to be congratulated on their production of the book. The print used is small, but it is readable, and it allows a lot of matter to be included in the space of 251 pages. The book is profuse with illustrations: these are well produced—much better so than in the last edition. Altogether, the book can be thoroughly recommended.—E. W. C. WILKINS.

THE SHAPE OF THE AEROPLANE. *James Hay Stevens. Hutchinson & Co. Ltd., London 1953. 302 pp. 280 illustrations by the author. 12s. 6d. net.*

Here is a book of standard dimensions but with an uncommonly high page-loading. By this we do not mean that the reading matter is heavy; the weight comes from the mass of information which has been packed into almost every line.

The author writes about and describes every aeroplane which has a claim to distinction, good or bad. He explains the why and wherefore of their existence and their main features, and garnishes his tale with scores of little-known facts. When he wishes to disparage he does not use the subtle phrase or gentle innuendo; if he holds a thing to be bad or ridiculous he says so in unequivocal and unvarnished language. Here and there he does some timely debunking; too many aeroplanes in their lifetime, and subsequently, have enjoyed golden reputations they did not merit.

"The Shape of the Aeroplane" was advertised in a Sunday newspaper (and probably elsewhere) as among "A fine selection of books for young people." But this should not deter any member of the Royal Aeronautical Society, whatever his age, from buying the book. There is nothing juvenile about it. Mr. Stevens is a master of lucid and mature exposition and explanation, and there is not a grown-up in the country who would find this book tediously elementary.

Even when his thoughts run ahead of his pen—as they did when he wrote (about the Beaufighter): "The wing was actually a strengthened version of the Beaufort's wing, as was also the undercarriage"—his meaning is still clear.

If the book has a fault, it can be found in the title, which is ten times too small. The book is encyclopædic in scope, but the design, dictated by the title, compels Mr. Stevens to reveal his seemingly boundless knowledge of the science, art and practice of aeronautics fragmentarily as and when an aeroplane gives him an opportunity.

When the book is reprinted, the publishers might consider adding as a sub-title: Anatomical and Historical

Studies of a Thousand Types "—or whatever the number happens to be. Then justice might be done to a work that obviously demanded its fee in toil and sweat.—S.E.V.

INSTRUMENT ENGINEERING, VOL. II. *Methods for Associating Mathematical Solutions with Common Forms. C. S. Draper, W. McKay, S. Lees. McGraw-Hill, New York, 1953. 827 pp. Index. 120s. net.*

This book is pleasantly printed by offset reproduction. It forms part of a major work in three volumes. The parts may be broadly classed as:—

Formulation of physical Problems in automatic control systems (Vol. I); the solutions of the equations of " servo " mathematics (Vol. II); the application of the foregoing to particular systems (Vol. III). Volume I was reviewed in this Journal in July 1953.

The reviewer made some constructive criticisms and found it necessary to express doubt whether Vol. I would be of use to the student or the practising engineer. Clearly undiscouraged by criticism the authors have toiled on to produce an even less digestible volume.

The material is essentially the mathematics of the theory of servo-mechanisms and the aim has been to present techniques and results in a form which is suitable for immediate use. This aim is utterly unfulfilled. The authors would not leave well alone and have added verisimilitude to an otherwise bald and unconvincing narrative by means of a truly elephantine notation. It is very well to seek a universal notation but the strain of managing variables with numerous suffixes, primes and other obstacles is intolerable and much of the work is unusable by anyone whose time is valuable.

The eight chapters which occupy the first 503 pages are concerned with matter that is the stock-in-trade of students of stability. Even so, the disguise just mentioned is used to such effect that it needs the patient study of many pages to follow the solution of equations like $x+ax=P\sin\omega t$. The first chapter is entitled, " Non-dimensionalisation of linear integro-differential equations"! and the eighth is called, " Methods of solving third and fourth order characteristic equations." The remaining chapters are: " Graphical methods of determining transient stability "; " Graphical calculation of steady state sinusoidal relating functions "; " Relating function forms from pulse function responses "; " Representations of solutions for differential equations with non-linear terms of the rate-determined step type "; " Numerical analysis by the number series transformation method." There is little in these later chapters that cannot be found elsewhere.

The M.I.T. Radiation Laboratory Series contains two volumes which atone as it were in anticipation, for this effort from the M.I.T. Instrumentation Laboratory. We refer to the " Theory of Servo-mechanisms " by James, Nichols and Phillips, and " Electronic Instruments " by Greenwood, Holdam and MacRae. These works contain stimulating accounts of more than half of the material of the chapters listed above. Later developments like the Evans " locus of roots " method are described with enthusiasm in William Bollay's 14th Wright Brothers lecture (*Journal of the Aeronautical Sciences*, September 1951). The content of 40 pages of the chapter on non-linear terms is available in Frazer, Duncan, and Collar's book. The 13 pages on phase plane plots cannot be understood without reference to works like Minorsky's " Introduction to Non-Linear Mechanics." The last chapter has been contributed by Albert Madwed of the M.I.T. and contains an account of his use and extension of Professor Tustin's time series method. This is taken from Dr. Madwed's D.Sc. thesis, a copy of which is held in the library of the Royal Aircraft Establishment.

In spite of these harsh words, the reviewer looks forward to the publication of Vol. III of the series. This may provide *a posterion* justification for the presentation used here.—P. T. FINK.

THE PHYSICS OF THE STRATOSPHERE. *R. M. Goody. Cambridge University Press 1954. 187 pp. Illustrated. 25s.*

In this latest volume in the Cambridge series of Monographs on Physics the author has summarised available information on many properties of the upper atmosphere which are of immediate importance to those concerned with high altitude aircraft or missile flight. The main section covers temperature, composition, ozone distribution and concentration, stratospheric winds and turbulence and the thermal energy of the earth's atmosphere.

The method adopted by the author is to sketch briefly the development of experimental study in each section and to note the more important results with comments on their probable applicability. He does not draw any particular conclusions himself but leaves the reader to do this after he has studied the many references which are given. It would be a brave—and possibly foolish—man who would at this stage of our knowledge attempt to lay down definite figures for many of the properties of the stratosphere under discussion.

The greatest test of any book is for it to be used. Goody's book can be used and has proved its value. It should be very useful to all who are interested in the physics or meteorology of the region of the earth's atmosphere up to 80 km.—D.C.S.

Additions to the Library

Bateman Manuscript Project. HIGHER TRANSCENDENTAL FUNCTIONS. Vol. I and Vol. II. McGraw Hill. 1953.

Benny, L. B. MATHEMATICS FOR STUDENTS OF ENGINEERING AND APPLIED SCIENCE. O.U.P. 1954.

*British Standards Institution. LETTER SYMBOLS, SIGNS AND ABBREVIATIONS. PART I: GENERAL. British Standard 1991. B.S.I. 1954.

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*Grambow, M. E. (*Editor*). AMERICAN AVIATION DIRECTORY. Vol. 15. No. 1. American Aviation. 1954.

Luecht, J. W. SOME FACTORS PERTAINING TO THE USE OF AIR-BREATHING PROPULSION FOR THE ACCELERATION OF HIGH-ALTITUDE SOUNDING ROCKETS AND OTHER LONG-RANGE BALLISTIC MISSILES (S.A.E. Preprint). S.A.E. 1953.

National Bureau Standards. TABLES OF CIRCULAR AND HYPERBOLIC SINES AND COSINES FOR RADIAN ARGUMENTS. U.S.G.P.O. 1953.

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*Reuss, K. F. JAHRBUCH DER LUFTFAHRT. 1954. Pohl & Co. 1954.

Rotblat, J. (*Editor*). ATOMIC ENERGY—A SURVEY. Taylor & Francis. 1954.

Thompson, H. MICROSCOPICAL TECHNIQUES IN METALLURGY. Pitman. 1954.

Weather Bureau. THE THUNDERSTORM. U.S.G.P.O. 1949.

Yates, R. F. MODEL JETS AND ROCKETS FOR BOYS. Laurie. 1954.

Young, J. F. MATERIALS AND PROCESSES, 2ND EDITION. Chapman Hall. 1954.

Items marked * are not available for loan.

Reports

AERODYNAMICS

Boundary Layer

Displacement effect of a three-dimensional boundary layer. F. K. Moore. N.A.C.A. Report 1124 (1953).

A method is described for determining the "displacement surface" of a known three-dimensional compressible boundary-layer flow in terms of the mass-flow defects associated with the profiles of the two velocity components parallel to the surface.—(1.1.0).

Cooling requirements for stability of laminar boundary layer with small pressure gradient at supersonic speeds. G. M. Low. N.A.C.A. T.N. 3103 (*March* 1954).

The amount of cooling required to stabilise the two-dimensional supersonic laminar boundary layer for all Reynolds numbers is calculated for flows with pressure gradients of a magnitude usually encountered over slender aerodynamic shapes. Only two-dimensional disturbances are treated in the stability calculations.—(1.1.2.4).

Statistical study of transition-point fluctuations in supersonic flow. J. C. Evvard, M. Tucker and W. C. Burgess. N.A.C.A. T.N. 3100 (*March* 1954).

The random movement of the transition point on a 10° cone at a free-stream Mach number of 3·12 was investigated by means of a large number of high-speed schlieren photographs. The distribution functions which statistically define the transition-point location were determined for a range of test-section Reynolds number and two levels of free-stream turbulence intensity. The intensity was varied by changing tunnel settling-chamber configurations. Temperature-recovery-factor distributions were also obtained.—(1.1.2.4).

Turbulent boundary-layer and skin-friction measurements in axial flow along cylinders at Mach numbers between 0·5 *and* 3·6. *D. R. Chapman and R. H. Kester. N.A.C.A. T.N.* 3097 (*March* 1954).

Direct-force measurements have been made of the average skin-friction coefficient for completely turbulent flow along the cylindrical portion of heat-insulated cone-cylinder bodies of revolution having overall fineness ratios of 10, 15 and 25. Numerous boundary-layer surveys were made. Reynolds numbers between 4 million and 32 million were investigated. At a Mach number of 2·0, data were obtained for different pressure distributions by distorting the flexible-plate walls of the wind tunnel.—(1.1.3).

Fluid Dynamics

Fall of cosmic dust from high altitudes. D. C. Collis. A.R.L. Aerodynamics Note 130 (*December* 1953).

The time taken by spherical particles of diameter 0·1 to 5·0 micron and density 3·5 gm./c.c. to fall from an altitude of 100 km. to the lower atmosphere is shown to be almost independent of their initial velocity. Neglecting the effect of agencies such as turbulent diffusion, the time of fall can be calculated from a knowledge of the terminal velocity as a function of altitude. The terminal velocity at all stages of the fall can be determined by using Millikan's empirical law of resistance to motion through a gas.—(1.4).

Impingement of water droplets on an ellipsoid with fineness ratio 5 *in axisymmetric flow. R. G. Dorsch, R. J. Brun and J. L. Gregg. N.A.C.A. T.N.* 3099 (*March* 1954).

Droplet trajectories about an ellipsoid of revolution with a fineness ratio of 5 were computed for incompressible axisymmetric air flow. From the computed droplet trajectories, the following impingement characteristics of the ellipsoid surface were obtained and are presented in terms of dimensionless parameters: (1) total rate of water impingement, (2) extent of droplet impingement zone, (3) distribution of impinging water, and (4) local rate of water impingement.—(1.4).

The use of tensor notation to develop characteristic equations of supersonic flow. C. N. H. Rock and R. C. Tomlinson. R. & M. 2632 (*March* 1949, *published* 1954).

The general equations of the steady motion of a non-viscous fluid are given in tensor notation. It is then assumed that one family of co-ordinate surfaces, $x^a = \text{constant}$, are characteristic surfaces, *i.e.* surfaces on which the transverse derivatives of the flow variables are not determined by their values on the surface itself. The condition for this is given by the relation $(u^a)^2 = a^2 g^{aa}$ which can be interpreted to give the well-known result that the velocity normal to the surface is sonic. The relation which must then hold between the variables on the surface itself is also determined (characteristic equation).—(1.4.1).

Velocity distribution on thin bodies of revolution at zero incidence in incompressible flow. S. Neumark. R. & M. 2814 (*July* 1950, *published* 1954).

A new method of determining velocity distribution on slender bodies of revolution in axial flow is expounded, analogous to the linear perturbation method widely used for slender symmetrical profiles in two dimensions. The proposed method leads to simple approximate formulae for velocity distribution on a body, once the equation of the meridian line is given, either in the form of a polynomial, or a square root of one. The new method avoids many inconveniences of the older procedures, and is much more rapid. Although theoretically applicable to bodies of small thickness only, it works with satisfactory accuracy up to quite considerable thickness ratios. It has been further improved by taking into account not only axial but also raidal velocity components, following a suggestion of Lighthill's supersonic theory. It may be easily applied to compressible subsonic flow.—(1.4.1).

Internal Flow

Design of two-dimensional channels with prescribed velocity distributions along the channel walls. J. D. Stanitz. N.A.C.A. Report 1115 (1953).

A general method of design is developed for two-dimensional unbranched channels with prescribed velocities as a function of arc length along the channel walls. The method is developed for both compressible and incompressible, irrotational, non-viscous flow and applies to the design of elbows, diffusers, nozzles, and so forth. Two types of compressible flow are considered: the general type, with the ratio of specific heats equal to 1·4, for example, and the linearised type, in which the ratio of specific heats is −1·0. Two methods of solution are used. In part I, solutions are obtained by relaxation methods and in part II, by a Green's function.—(1.5.1).

Airflow characteristics of "aerofin" cooling elements placed in a right angle bend mitre. H. J. Higgs and H. G. Darling. A.R.L. Aerodynamics Note 118 (*July* 1953).

The pressure loss characteristics and velocity distribution are presented for aerofin tubes and sheet metal corner vanes placed in the right angle bend of a circular duct.—(1.5.1).

The performance of the 108 *compressor fitted with low stagger free vortex blading. D. V. Foster. A.R.C. Current Paper* 144 (*June* 1952).

A large three-stage compressor is described which has been designed for detail three-dimensional flow investigations. Particular attention has been paid to the accuracy of measurement on the rig and it is shown that the main errors are due to the unsteady nature of the flow and to speed fluctuations. Both of these factors are, however, considerably less than those normally experienced on compressor test work.—(1.5.2.1 × 27.1).

NOTE:—*The figures in parentheses at the end of each Summary are for office use only.*

One-dimensional analysis of choked-flow turbines. R. E. English and R. H. Cavicchi. N.A.C.A. Report 1127 (1953).

It is indicated that the area ratios and equivalent blade speed are the controlling factors in the design and operation of high work output turbines. Six criteria are stated that will aid in establishing from test data of multi-stage turbines which blade rows are choked and which are not. The variation in internal flow conditions with operating conditions for a turbine equipped with an adjustable stator is determined for one application of stator adjustment.—(1.5.3 × 27.1).

Application of a channel design method to high-solidity cascades and tests of an impulse cascade with 90° of turning. J. D. Stanitz and L. J. Sheldrake. N.A.C.A. Report 1116 (1953).

A technique is developed for the application of a channel design method to the design of high-solidity cascades with prescribed velocity distributions as a function of arc length along the blade-element profile. The technique applies to both incompressible and subsonic linearised compressible (ratio of specific heats equal to −1·0), non-viscous, irrotational, fluid motion.—(1.5.4).

Stability and Control

A progress report on the theoretical aspects of limit control. D. Bushaw. Stevens Institute of Technology. Report No. 427 (*November* 1951.—(1.8.0).

Some effects of frequency on the contribution of a vertical tail to the free aerodynamic damping of a model oscillating in yaw. J. D. Bird, L. R. Fisher and S. M. Hubbard. N.A.C.A. Report 1130 (1953).

The directional damping and stability of a fuselage-vertical-tail model oscillating freely in yaw were measured at a Mach number of 0·14 and compared with the damping and stability obtained by consideration of the effects of unsteady lift. The effects of vertical-tail aspect ratio and compressibility as predicted by the theoretical treatments are discussed in relation to the experimental stability characteristics obtained by the free-oscillation and by the curved-flow procedures.—(1.8.1).

Thermo-Aerodynamics

Experimental investigation of sublimation of ice at subsonic and supersonic speeds and its relation to heat transfer. W. D. Coles and R. S. Ruggeri. N.A.C.A. T.N. 3104 (*March* 1954).

An experimental investigation was conducted in the 3·84-by 10-inch tunnel to determine the mass transfer by sublimation, heat transfer, and skin friction for an iced surface at subsonic and supersonic speeds.—(1·9).

Heat transfer, diffusion, and evaporation. W. Nusselt. N.A.C.A. T.M. 1367 (*March* 1954).

The general similarity of heat and mass transfer (diffusion) processes is discussed, with particular reference to the lack of complete identity of the relations governing the two phenomena. It is indicated that, for example, the boundary conditions in the two cases at the surface of a body will not be the same. The correct equation of diffusion is given for various simple cases. Generalised relations for combined heat and mass transfer are then evolved for particular situations, comparisons being made among several different approaches to the problem. Finally, the effect of a buoyancy force field on the generalised relations is considered, with special reference to the evaporation of water. —(1.9).

Wings and Aerofoils

Theoretical prediction of pressure distributions on nonlifting airfoils at high subsonic speeds. J. R. Spreiter and A. Alksne. N.A.C.A. T.N. 3096 (*March* 1954).

Theoretical pressure distributions on non-lifting circular-arc aerofoils in flows with high subsonic free-stream velocity are found by determining approximate solutions of an integral equation for transonic flow proposed by Oswatitsch. Solutions are obtained using an iteration process which differs from the classical methods in that the quadratic nature of the integral equation is recognized.—(1.10.1.1).

A study of inviscid flow about airfoils at high supersonic speeds. A. J. Eggers, C. A. Syverton and S. Kraus. N.A.C.A. Report 1123 (1953).

Steady flow about curved aerofoils is investigated analytically, first assuming air behaves as an ideal gas, and then assuming it behaves as a thermally perfect, calorically imperfect gas. Conclusions are drawn from the study.—(1.10.1.1).

*On the theory of the oscillating wing in two-dimensional subsonic flow. E. Van Spiegel and A. I. Van de Vooren. N.L.L. Report F.*142 (*November* 1953).

A short survey of the theory of Timman and van de Vooren for the oscillating wing is given. The pressure distribution is also derived by using a method which is a generalisation of Hofsommer's method for the incompressible case. It is proved that the solution of Timman satisfies the integral equation of Possio. In the Appendix the kernel of Possio's equation has been reduced to a form given by Kussner, but the expression found in this report deviates slightly from that of Kussner.—(1.10.1.1).

*The aerodynamic forces and moments on an oscillating aerofoil with control-surface between two parallel walls. E. M. de Jager. N.L.L. Report F.*140 (*October* 1953).

Formulae are given for the aerodynamic forces and moments of an oscillating aerofoil with control surface in two-dimensional incompressible flow, while the aerofoil is placed between two parallel walls. The corrections of the aerodynamic derivatives, due to the presence of the walls, are plotted against the reduced frequency for three values of the ratio between chord length and distance of the walls; for the control surface derivatives two values have been assumed for the control surface chord.—(1.10.1.1).

A design study of leading-edge inlets for unswept wings. R. E. Dannenberg. N.A.C.A. T.N. 3126 (*March* 1954).

A method is presented for calculating the profile co-ordinates for an inlet to be placed in the leading edge of an aerofoil from formulas. The method includes an application of the principles of thin-aerofoil theory which permits the change in velocity distribution caused by a variation in inlet profile to be calculated. Wind-tunnel tests of leading-edge inlets in an aerofoil having the N.A.C.A. 63_1-012 section were made to evaluate the effects of the inlets on the aerodynamic characteristics of the aerofoil.—(1.10.1.2 × 1.5.1).

Aerodynamics of slender wings and wing-body combinations having swept trailing edges. H. Mirels. N.A.C.A. T.N. 3105 (*March* 1954).

A general method, based on two-dimensional cross-flow concepts, is presented for obtaining the lift and moments on highly swept wings. Emphasis is placed on obtaining solutions for wings having swept trailing edges. The method is applicable for all problems where the velocity boundary conditions can be made homogeneous by differentiation in the streamwise or spanwise directions. Lift, roll, and pitch solutions, for highly swept wings, are presented. Both direct problems and inverse problems are considered. Applications of the method for the solution of wing-body problems and unsteady two-dimensional incompressible flow problems are also indicated.—(1.10.1.2).

*An approach to lifting surface theory. A. I. Van de Vooren. N.L.L. Report F.*129 (*June* 1953).

By using a series expansion for the chordwise vorticity distribution, a set of integral equations for the coefficients in this expansion is obtained. In the simplest case this set of integral equations simplifies into Weissinger's equation. All chordwise integrations can be performed by aid of pivotal points for which the best positions are derived. A function, given explicitly in the report, must be evaluated in these points.—(1.10.1.2).

Critical Mach numbers for thin untapered swept wings at zero incidence. S. Neumark. R. & M. 2821 (*November* 1949, *published* 1954).

In this paper, which is a continuation of two earlier ones R. & Ms. 2713 and 2717) the subsonic flow past untapered swept wings, at zero incidence, is further investigated using linear theory. Methods for calculating "lower" and

"upper" critical Mach numbers are given, the solution of the main problem being preceded by a short analysis of critical Mach numbers for the simpler cases of infinite wings (straight, sheared and yawed).—(1.10.1.2).

Theoretical supersonic force and moment coefficients on a sideslipping vertical—and horizontal—tail combination with subsonic leading edges and supersonic trailing edges. F. S. Malvestuto. N.A.C.A. T.N. 3071 (*March* 1954).

Formulas are obtained by means of linearised flow theory for the lateral force due to sideslip, the yawing moment due to sideslip, and the rolling moment due to sideslip for a tail arrangement composed of a vertical triangular surface attached to a horizontal triangular surface. The leading edges of the surfaces are subsonic; the trailing edges, supersonic. A series of design charts are presented for rapid estimates of the stability derivatives considered.—(1.10.1.2 × 1.8.1.2).

Effects of subsonic Mach number on the forces and pressure distributions on four N.A.C.A. 64A-series airfoils at angles of attack as high as 28°. *L. S. Stivers, N.A.C.A. T.N.* 3162 (*March* 1954).

A high-speed wind-tunnel investigation has been made of the N.A.C.A. 64A010, 64A410, 64A006 and 64A406 aerofoil sections at angles of attack as high as 28°. Pressure-distribution, lift, drag, and moment characteristics were determined for each aero-foil section at Mach numbers ranging from 0·30 to about 0·93, with corresponding Reynolds numbers varying from approximately $0{\cdot}9\times10^5$ to $1{\cdot}9\times10^6$. The $a=0{\cdot}8$ (modified) mean line was employed for the cambered aerofoil sections.—(1.10.2.1).

Investigation of a slat in several different positions on an N.A.C.A. 64A010 airfoil for a wide range of subsonic Mach numbers. J. A. Axelson and G. L. Stevens. N.A.C.A. T.N. 3129 (*March* 1954).

An investigation of a slat in several different positions on an N.A.C.A. 64A010 aerofoil has been conducted over a Mach number range from 0·25 to 0·85, corresponding to a Reynolds number range from 3·4 million to 8·1 million. Section force coefficients and chordwise distributions of pressure and resultant loading are presented for several different locations of the slat.—(1.10.2.1).

The effectiveness at high subsonic Mach numbers of a 20-per cent.-chord plain trailing-edge flap on the N.A.C.A. 65-210 airfoil section. L. S. Stivers. N.A.C.A. T.N. 3127 (*March* 1954).

The lift-control characteristics of a 20-per cent.-chord plain trailing-edge flap on the N.A.C.A. 65-210 aerofoil section have been analysed for Mach numbers from 0·30 to 0·875, utilising section lift-coefficient data. In addition, the relative merits of the plain flap, a spoiler, and a dive-recovery flap employed separately as a lift-control device on the N.A.C.A. 65-210 aerofoil section have been determined from increments of section lift, drag, and pitching-moment coefficients which accompanied the deflection of each control device. The Reynolds number of the data varied with Mach number from about 1×10^6 to 2×10^6.—(1.10.2.1).

Half-model tests of two small aspect ratio delta wings at subsonic, transonic and supersonic speeds. J. Lukasiewicz, J. D. Stewart and J. G. LaBerge. N.A.E. L.R.-92 (*January* 1954).

The aerodynamic characteristics of two small aspect-ratio ($A=2{\cdot}12$ and $2{\cdot}77$) delta wings were determined at Mach numbers from 0·6 to 2·5 and Reynolds numbers (based on m.a.c.) from 1·0 to 1·6 million in the 10-inch and 30-inch wind tunnels using half-models mounted on the tunnel wall.—(1.10.2.2).

Zero lift drag measurements on swept wings at transonic and supersonic speeds using the ground-launched rocket-boosted model technique. T. Lawrence and C. Kell. A.R.C. Current Paper—145 (*May* 1952).

This is mainly a documentary record of drag measurements on 14 swept wings varying in plan form from deltas to swept untapered wings, and from 4 per cent. to 10 per cent. thick. The Reynolds number of the tests was 7×10^6 per foot chord at $M=1{\cdot}0$. The results are compared with theory for wings of double wedge section, and an attempt is made to check the validity of the supersonic similarity laws.—(1.10.2.2).

Testing and Instruments

Ion tracer technique for air speed measurements at low densities. W. B. Kunkel and L. Talbot. N.A.C.A. T.N. 3177 (*March* 1954).

A study has been made of the aerodynamic and production detection problems associated with ion tracer velocity measurement techniques, with particular emphasis on those aspects unique to low-density gas dynamics. A critical survey was made of the various techniques which have been employed and specific suggestions are offered relative to the successful use of ion tracer velocity measuring methods at low densities. A description is also included of some experimental work done with an ion-pulse air-speed indicator developed by the Ames Aeronautical Laboratory Low Pressures Group of the National Advisory Committee for Aeronautics for use in low-density wind tunnels.—(1.12).

AIRCRAFT DESIGN AND CONSTRUCTION

An analytical and experimental study of the transient response of a pressure-regulating relief valve in a hydraulic circuit. H. Gold and E. W. Otto. N.A.C.A. T.N. 3102 (*March* 1954).

The transient response of the pressure-regulating relief valve in a hydraulic circuit is analysed by means of an electrical analogy of the hydraulic circuit. Measurements of the transient response of a hydraulic relief valve are presented and are compared with responses calculated from the differential equation of the equivalent electric network.—(4.2.3).

FUELS AND LUBRICANTS

Friction and wear investigation of molybdenum disulfide. II—effects of contaminants and method of application. M. B. Peterson and R. L. Johnson. N.A.C.A. T.N. 3111 (*March* 1954).

An experimental study was conducted using a low-speed kinetic-friction apparatus to show the effects of method of application and contaminants on the lubricating characteristics of molybdenum disulfide MoS_2.—(14.3).

INSTRUMENTS AND EQUIPMENT

Vibration and flutter of aircraft aerials. W. H. Johnson. Current Paper—146 (*December* 1953).

Fatigue failures of blade and whip aircraft aerials have been investigated and it is shown that stalling flutter and mechanically-excited vibration have both contributed in large measure to the failures. All the aerial types involved possess considerable flexibility and very low internal damping.—(18.1).

MATERIALS

Effect of heat treatment upon the microstructure and hardness of a wrought cobalt-base alloy Stellite 21 (*AMS* 5385). *F. J. Clauss and J. W. Weeton. N.A.C.A. T.N.* 3107 (*March* 1954).

An investigation has been made to study the response of wrought Stellite 21 to heat treatment. A period of 72 hours at 2,250°F. was sufficient for effective solution treatment of this alloy.—(21.5).

Relation of microstructure to high-temperature properties of a wrought cobalt-base alloy Stellite 21 (*AMS* 5385). *F. J. Clauss and J. W. Weeton. N.A.C.A. T.N.* 3108 (*March* 1954).

An investigation was conducted to determine the effects of microstructure on stress-rupture life of heat-treated wrought Stellite 21.—(21.5).

An investigation of lamellar structures and minor phases in eleven cobalt-base alloys before and after heat treatment. J. W. Weeton and R. A. Signorelli. N.A.C.A. T.N. 3109 (*March* 1954).

An investigation of the occurrence and nature of lamellar structures and minor phases of eleven cobalt-base high-temperature alloys (rolled, cast, and high-carbon Stellite 21, 422-19, X-63, 6059, 61, X-40, S-816, I-336, and J).—(21.5).

A further investigation of the effect of surface finish on fatigue properties at elevated temperatures. R. L. Ferguson. N.A.C.A. T.N. 3142 (*March* 1954).

An investigation was conducted to evaluate the effects of surface roughness on fatigue properties of low carbon N-155 alloy with a grain size of A.S.T.M. 6 and of S-816 alloy with a grain size of A.S.T.M. 6 to 7. Fatigue studies were conducted at 80°, 1,200°, 1,350° and 1,500°F. In addition, an investigation of the effect of surface abrasion upon the character of residual stresses and of the effect of time and temperature upon the relief of these stresses was conducted.—(21.2.1).

Further studies of the mechanism by which hydrogen enters metals during chemical and electrochemical processing. L. D. McGraw, W. E. Ditmars, C. A. Snavely and C. L. Faust. N.A.C.A. T.N. 3164.

This investigation was a continuation of efforts to develop fundamental concepts which would be useful in preventing or alleviating hydrogen damage to steel. N.A.C.A. Technical Note 2696 presented a new mechanism which postulates that the entry of hydrogen into metals from chemical and electro-chemical action occurs when hydrogen-metal alloy formation is simultaneous with discharge (or release) of hydrogen atoms. In the present work the phenomena relating to hydrogen entry into steel were studied further.—(21.5).

MISSILES

*Contribution a l'étude de la transmission infrarouge de la basse atmosphère. P. Barchewitz, G. Amat and C. Rossetti. Publications Scientifiques et Technicques. No. B.S.T.*116 (1954). —(24).

Measurements of pressure and temperature for appraisal of the temperature method of air speed calibration in the lower stratosphere. L. J. Lina. N.A.C.A. T.N. 3075 (*March* 1954).

An investigation was made to determine whether the temperature and pressure conditions in the lower stratosphere would meet the requirements of the temperature method for accurate calibration of air-speed installations. Measurements of temperature and pressure were made in flights of a swept-wing fighter aeroplane on four clear days in March and April 1953 near Langley Field, Virginia.—(24).

POWER PLANTS

Factors influencing the optimum aerodynamic design of cooled turbines. G. F. C. Rogers. A.R.C. Current Paper 155 (*July* 1952).

Because of the losses in performance which increase with the rate of heat extraction required for turbine cooling, it is desirable to know what range of aerodynamic designs is associated with low values of this quantity. Different aerodynamic designs of turbine, all passing the same mass flow and having approximately the same disc and blade stresses, have been compared on the basis of the ratio of heat extraction rate to work output.—(27.1).

A thermodynamic study of the turbine-propeller engine. B. Pinkel and I. M. Karp. N.A.C.A. Report 1114 (1953).

Equations and charts are presented for computing performance parameters of a turbine-propeller engine for any given set of operating conditions and component efficiencies. To illustrate some of the turbine-propeller system performance characteristics, the total thrust horsepower per unit mass rate of air flow and the specific fuel consumption are presented for a wide range of flight engine design operating conditions and given design component efficiencies. The performance of a turbine-propeller engine containing a matched set of components is presented for a range of engine operating conditions.—(27.1).

Analog study of interacting and non-interacting multiple-loop control systems for turbojet engines. G. J. Pack and W. E. Phillips. N.A.C.A. T.N. 3112 (*March* 1954).

An analog investigation of several turbo-jet control configurations was made. Both proportional and proportional-plus-integral controllers were studied, and compensating terms for engine interaction were added to the control system. Data were taken on the stability limits and the transient responses of these various configurations. Analytical expressions in terms of the component transfer functions were developed for the configurations studied and the optimum form for the compensation terms was determined.—(27.1).

STRUCTURES

LOADS

The effect of rolling on fin-and-rudder loads in yawing manoeuvres. D. R. Puttock. A.R.C. Current Paper 153 (*January* 1953).

Exact solutions are derived for angle of sideslip and fin-and-rubber loads for an aircraft performing two yawing manoeuvres induced by the rudder. Angles of sideslip and fin-and-rudder loads are then calculated for three selected aircraft and compared with results obtained by a simplified method in which rolling motion is neglected. Further calculations are made using a modified method in which the coefficients of the response formulae of the simplified method have been adjusted to take some account of rolling.—(33.1.1).

Effects of panel flexibility on natural vibration frequencies of box beams. B. Budiansky and R. W. Fralich. N.A.C.A. T.N. 3070 (*March* 1954).

Effects of local panel oscillations on bending and torsional vibrations of box beams with flexible covers and webs are investigated. Theoretical analyses of simplified models are made in order to shed light on the mechanism of coupling between local and overall vibrations and to derive results that can be used to estimate the coupling effects in box beams.—(33.1.2).

A preliminary investigation of the effects of gusty air on helicopter-blade bending moments. J. W. Jewel and P. J. Carpenter. N.A.C.A. T.N. 3074 (*March* 1954).

A preliminary investigation has been made at the Langley helicopter test tower to determine the bending moments excited on typical helicopter rotor blades in quiet and gusty air.—(33.1.2).

Fatigue investigation of full-scale transport-aeroplane wings. Summary of constant-amplitude tests through 1953. *M. J. McGuigan, D. F. Bryan and R. E. Whaley. N.A.C.A. T.N.* 3190 (*March* 1954).

Results are presented of constant-amplitude fatigue tests on the wings of C-46 aeroplanes. Eight complete wings were tested at four alternating load levels ranging from 7·5 to 22 per cent. of the design ultimate load factor. All tests were conducted with a 1 *g* mean load, which is about 22 per cent. of the design ultimate load factor. The rate and manner of fatigue-crack propagation was investigated, and the effect of the magnitude of load on these items was determined. The scatter in fatigue life of the wings was analysed statistically. An appendix by M. H. Levin presents information on the use of bonded wires to detect small fatigue cracks.—(33.1.2 × 33.2.3.1).

THEORY AND ANALYSIS

*Buckling of a right angled isosceles triangular plate in combined compression and shear. (Perpendicular edges simply supported, hypotenuse clamped.) W. H. Wittrick. A.H.L./SM.*220 (*November* 1953).

The problem considered is that of the buckling of a right-angled isosceles triangular plate, simply supported along its two perpendicular edges and clamped along the hypotenuse. The applied load is a combination of uniform shear along the perpendicular edges, and uniform compression in all directions. An interaction curve between the shearing and compressive buckling stresses is derived.—(33.2.4.7.6).

*A shell analogy for framed domes. L. H. Mitchell. A.R.L./SM.*208 (*December* 1953).

The accuracy of the shell analogy for framed domes has been investigated by analysing the "Dome of Discovery." A relaxation solution to this problem has been published and it is shown here that the shell analogy gives results of sufficient accuracy for design purposes, with considerably less labour.—(33.2.4.11.3).

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RELIABILITY

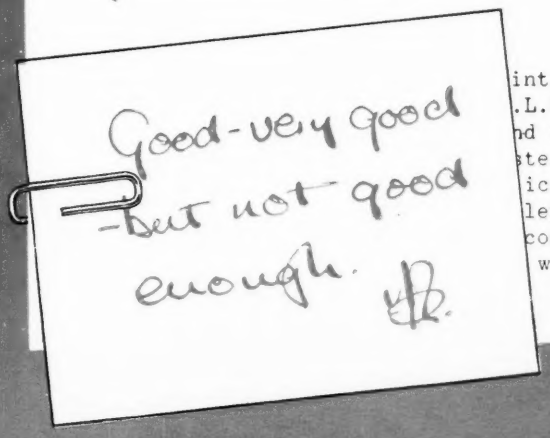
HYDRAULIC COMPONENTS

Airframe hours flown	1450
Aircraft in service	12
No. of hydraulic components	1000
Components confirmed faulty	2

LANDING GEAR

Total number of landings to date	22,572

An oleo has never failed on service causing a collapse.



int those concerned with the
.L.W. system in the [redacted]
nd landings of the [redacted] fleet
sted the hydraulic system under
ic components are overhauled
lent to 2,400 flying hours, but
components are the original
were new.

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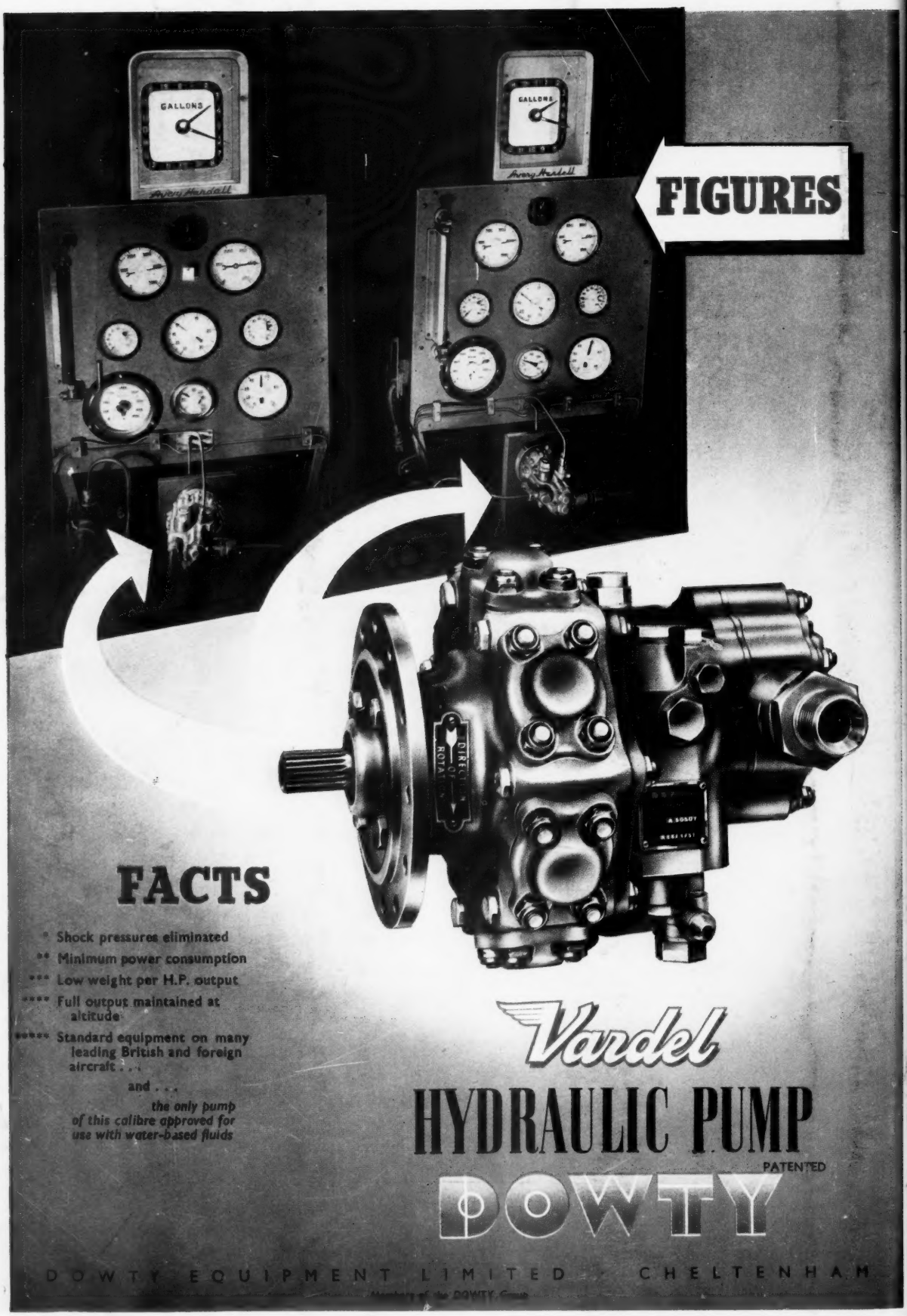

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